# 1 INTRODUCTION

## LEARNING OBJECTIVES

- Concept of Mechanics and Applied Mechanics
- Importance and Necessity of Applied Mechanics
- Branches of Mechanics
- Concept of Rigid Body
- Definitions of Terms Used in Mechanics
- System of Units Used in Mechanics
- Resolution of a Force
- Laws of Mechanics

## 1.1 CONCEPT OF MECHANICS AND APPLIED MECHANICS

Mechanics is that branch of science which deals with the behaviour of a body when the body is at rest or in motion. The mechanics may be divided into Statics and Dynamics. The branch of science, which deals with the study of a body when the body is at rest, is known as Statics while the branch of science which deals with the study of a body when the body is in motion, is known as Dynamics. Dynamics is further divided into kinematics and kinetics. The study of a body in motion, when the forces which cause the motion are not considered, is called kinematics and if the forces are also considered for the body in motion, that branch of science is called kinetics. The classification of Applied Mechanics are shown in Fig. 1.1 below.

APPLIED MECHANICS

1. Static
(body is at rest)

2. Dynamics
(body is in motion)

(i) kinematics
(body is in motion. The forces which cause motion, are not considered)

(i) kinematics
(body is in motion. Forces which cause motion, are considered)

**Fig. 1.1**

Applied mechanics deals with the application of principles and laws of mechanics to the practical engineering problems.

Actually mechanics is a science which is based on a systematic understanding and gathering of the facts, laws and principles governing natural phenomenon. Applied mechanics is an art of utilisation of the established facts, laws and principles to create certain desired phenomenon,.

**Note.** Statics deals with equilibrium of bodies at rest, whereas dynamics deals with the motion of bodies and the forces that cause them.

## 1.2 IMPORTANCE AND NECESSITY OF APPLIED MECHANICS

These days, the young engineers should have sound knowledge of fundamental subjects such as mechanics. They must have sound understanding of the fundamental principles that apply and be familiar with various general methods of solution of engineering problems rather than proficient in the use of any one. The study of applied mechanics makes the young engineers to build a strong foundation, to acquaint them with as many general methods of solution as possible and to illustrate the application of these methods to practical engineering problems.

Applied mechanics deals with the application of principles and laws of mechanics to the practical engineering problems.

## 1.3 BRANCHES OF MECHANICS

The mechanics is the branch of science which deals with the physical state of rest or motion of bodies under the action of forces. Depending upon the nature of the body involved, the mechanics can be divided into:

(i) Mechanics of rigid bodies (known as Applied Mechanics)
(ii) Mechanics of deformable bodies (known as Mechanics of solids or strength of materials)
(iii) Mechanics of fluids.

BRANCHES OF MECHANICS

- Applied Mechanics (Mechanics of Rigid bodies)
- Mechanics of Solids
- Mechanics of Fluids

**Fig. 1.1(a)** shows the branches of mechanics

## 1.4 CONCEPT OF RIGID BODY

Rigid bodies are those bodies which do not deform under the action of applied forces. The distance between any two points remains constant, when body is subjected to external forces.

Under the action of loads or external forces the physical bodies deform, although slightly. But in many situation this deformation is negligibly small to affect the results. So, the assumption of a rigid body shall mean that the body does not deform or the distances between any two points of the body does not change under the action of applied forces. Applied mechanics is the mechanics of rigid body.

## 1.5 DEFINITIONS OF TERMS USED IN MECHANICS

**1.5.1. Vector Quantity.** A quantity which is completely specified by magnitude and direction, is known as a vector quantity. Some examples of vector quantities are: velocity, acceleration, force and momentum. A vector quantity is represented by means of a straight line with an arrow as shown in Fig. 1.2. The length of the straight line (i.e., AB) represents the magnitude and arrow represents the direction of the vector. The symbol $\overrightarrow{AB}$ also represents this vector, which means it is acting from A to B.

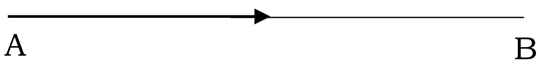



**Fig. 1.2** Vector Quantity

**1.5.2. Scalar Quantity.** A quantity, which is completely specified by magnitude only, is known as a scalar quantity. Some examples of scalar quantity are : mase,length, time and temperature.

**1.5.3. A Particle.** A particle is a body of infinitely small volume (or a particle is a body of negligible dimensions) and the mass of the particle is considered to be concentrated at a point. Hence a particle is assumed to a point and the mass of the particle is concentrated at this point.

**1.5.4. Law of Parallelogram of Forces.** The law of parallelogram of forces is used to determine the resultant* of two forces acting at a point in a plane. It states, “If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point."

Let two forces $P$ and $Q$ act at a point $O$ as shown in Fig. 1.3. The force $P$ is represented in magnitude and direction by $OA$ whereas the force $Q$ is presented in magnitude and direction by $OB$. Let the angle between the two forces be '$\alpha$'. The resultant of these two forces will be obtained in magnitude and direction by the diagonal (passing through *O*) of the parallelogram of which *OA* and *OB* are two adjacent sides. Hence draw the parallelogram with *OA* and O*B* as adjacent sides as shown in Fig. 1.4. The resultant *R* is represented by *OC* in magnitude and direction.

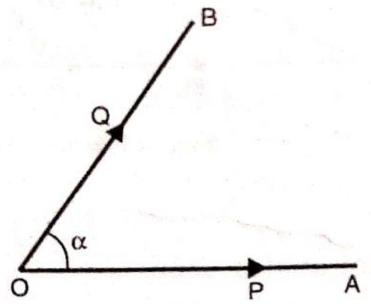



**Fig. 1.3**

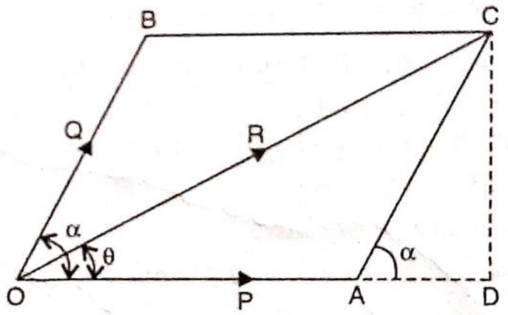



**Fig. 1.4**

**Magnitude of Resultant (R)**

From *C* draw *CD* perpendicular to *OA* produced.

Let $\alpha$ = Angle between two forces *P* and *Q* = $\angle AOB$

Now $\angle DAC = \angle AOB$ (Corresponding angles)

In parallelogram *OACB, AC* is parallel and equal to *OB*.

$\therefore \quad AC = Q$

In triangle ACD

$$AD = AC \cos \alpha = Q \cos \alpha$$

and $$CD = AC \sin \alpha = Q \sin \alpha$$

In triangle OCD

$$OC^2 = OD^2 + DC^2$$

But $OC = R,\ OD = OA + AD = P + Q \cos \alpha$

and $DC = Q \sin \alpha.$

$$\therefore \quad R = (P + Q \cos \alpha)^2 + (Q \sin \alpha)^2$$

$$= P^2 + Q^2 \cos^2 \alpha\ 2PQ \cos \alpha + Q^2 \sin^2 \alpha$$

$$= P^2 + Q^2 (\cos^2 \alpha + \sin^2 \alpha)\ 2PQ \cos \alpha$$

$$= P^2 + Q^2 + 2PQ \cos \alpha \qquad (\therefore \quad \cos^2 \alpha + \sin^2 \alpha = 1)$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ \cos} \qquad \ldots(1.1)$$

Equation (1.1) gives the magnitude of resultant force R.

**Direction of Resultant**

Let θ = Angle made by resultant with OA.

Then from triangle OCD

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P+Q\cos\alpha}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right) \qquad \ldots(1.2)$$

Equation (1.2) gives the direction of resultant (R).

The magnitude of resultant can also be obtained by using since rule [In triangle OAC, OA = P, AC = Q, OC = R, angle OAC = (180 – $\alpha$), angle ACO = 180 - [θ + 180 – $\alpha$] = ($\alpha$ – θ)

$$\frac{\sin\theta}{AC} = \frac{\sin(180-\alpha)}{OC} = \frac{\sin(\alpha-\theta)}{OA}$$

$$\frac{\sin\theta}{Q} = \frac{\sin(180-\alpha)}{R} = \frac{\sin(\alpha-\theta)}{P}$$

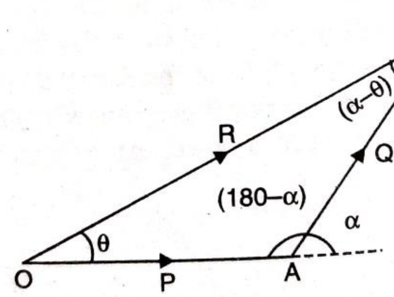


Fig. 1.4($\alpha$)

Two cases are important.

**1st case.** If the two forces $P$ and $Q$ act at right angles, then

$$\alpha = 90°$$

From Equation (1.1) we get the magnitude of resultant as

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos} = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$$

$$= \sqrt{P^2 + Q^2} \qquad (\because \cos 90° = 0) \qquad \ldots(1.2A)$$

From equation (1.2), the direction of resultant is obtained as

$$\therefore \quad \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$$

$$= \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right) = \tan^{-1}\frac{Q}{P} \quad (\because \sin 90° = 1 \text{ and } \cos 90° = 0)$$

**2nd case.** The two forces P and Q are equal and are acting at an angle $\alpha$ between them. Then the magnitude and direction of resultant is given as

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{P^2 + P^2 + 2P \times P \times \cos\alpha}$$

$$(\because P = Q)$$

$$= \sqrt{2P^2 + 2P^2\cos\alpha} = \sqrt{2P^2(1+\cos\alpha)}$$

$$= \sqrt{2P^2 \times 2\cos^2 \frac{\alpha}{2}} \qquad (\because 1 + \cos\alpha = 2\cos^2 \frac{\alpha}{2})$$

$$= \sqrt{4P^2 \times 2\cos^2 \frac{\alpha}{2}} \quad 2P\cos\frac{\alpha}{2}$$

and $$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right) = \tan^{-1}\frac{Q\sin\alpha}{P+P\cos\alpha} \quad (\because P = Q)$$

$$= \tan^{-1}\frac{Q\sin\alpha}{P(1+\cos\alpha)} = \tan^{-1}\frac{Q\sin\alpha}{1+\cos\alpha}$$

$$= \tan^{-1}\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}} \qquad (\because \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2})$$

$$= \tan^{-1}\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \tan^{-1}\left(\tan\frac{\alpha}{2}\right) = \frac{\alpha}{2} \qquad \ldots(1.4)$$

It is not necessary that one of two forces, should be along $x$-axis. The forces P and Q may be in any direction as shown in Fig. 1.5. If the angle between the two forces is '$\alpha$', then their resultant will be given by equation (1.1). The direction of the resultant would be obtained from equation (1.2). But angle will be the angle made by resultant with the direction of P.

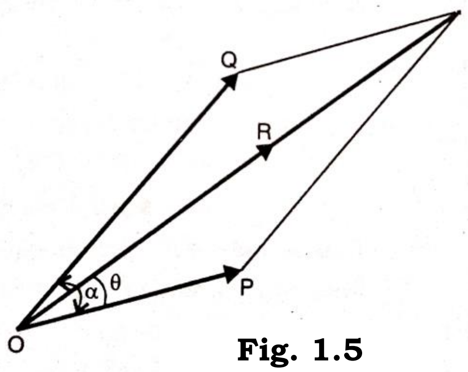



**Fig. 1.5**

**1.5.5. Law of Triangle of Forces.** It states that, "if three forces acting at a point be represented in magnitude and direction by the three sides of a triangle, taken in order, they will be in equilibrium."

1.5.6. Lami's Theorem. It states that, "If there forces acting at a point are in equilibrium are shown in Fig.16

Let α = Angle between force P and Q

β = Angle between force Q and R

γ = Angle between force R and P.

Then according to Lami's theorem P α sine of angle between Q and R α sine β.

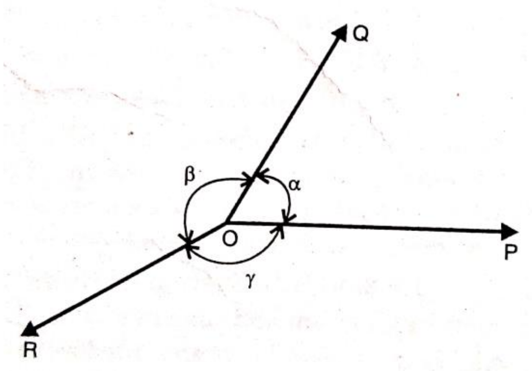



**Fig. 1.6**

$$\therefore \qquad \frac{P}{\sin\beta} = \text{constant}$$

Similarly, $\frac{Q}{\sin\gamma}$ = constant and $\frac{R}{\sin\alpha}$ = constant

or $$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

**Proof of Lumi's Theorem.** 'The three forces acting on a point, are in equilibrium and hence they can be represented by the three sides of the triangle taken in the same order. Now draw the force triangle as shown in Fig. 1.6 (*a*).

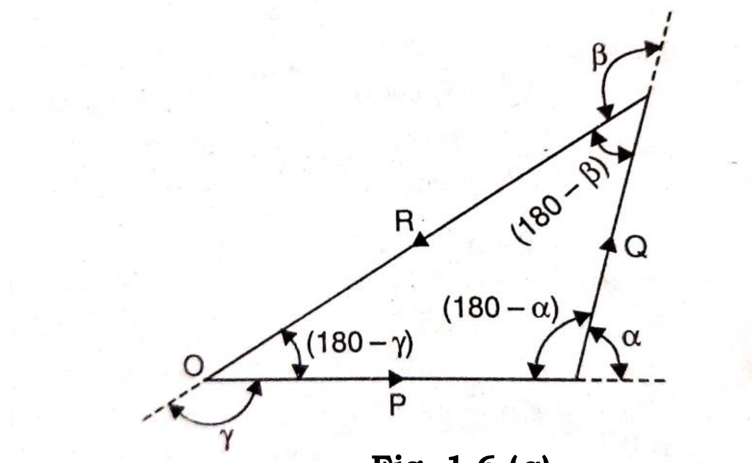



**Fig. 1.6 (*a*)**

Now applying sine rule, we get

$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\gamma)} = \frac{R}{\sin(180-\alpha)}$$

This can also be written

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

This is name equation (1.5).

**Note.** All the three forces should be acting either towards the point or away from the point.

## 1.6 SYSTEM OF UNITS USED IN MECHANICS

The following system of units are mostly used:

1. C.G.S. (i.e., Centimetre-Gram-Second) system of units.
2. M.K.S. (i.e., Metre-Kilogram-Second) system of units.
3. S.I. (i.e., International) system of units.

**1.6.1. C.G.S. System of Units.** In this system, length is expressed in centimemtre mass in gram and time in second. The unit of force in this system in dyne. Which is define as the force acting on a mass of one gram and producing an acceleration of one centimetre per second square.

**1.6.2. M.K.S. System of Units.** In this system, length is expressed in metre, mass in kilogram and time in second. The unit of force in this system is expressed as kilogram force and is represented as kgf.

**1.6.3. S.I. System of Units.** S.1. is abbreviation for The System International Units'. It is also called the International System of Units. In this system length is expressed in metre mass in kilogram and time in second. The unit of force in this system is Newton and is represented N. Newton is the force acting on a mass of one kilogram and producing an acceleration of one metre per second square. The relation between newton (N) and dyne is obtained as

$$\text{One Newton} = \text{One kilogram mass} \times \frac{\text{One meter}}{s^2}$$

$$= 1000 \text{ gm} \times \frac{100 \text{ cm}}{s^2} \qquad (\because \text{one kg} = 1000 \text{ gm})$$

$$= 1000 \times 100 = \frac{\text{gm} \times \text{cm}}{s^2}$$

$$= 10^5 \text{ dyne} \qquad \left\{\because \text{dyne} = \frac{\text{gm} \times \text{cm}}{s^2}\right\}$$

When the magnitude of forces is very large, then the unit of force like kilo-newton and mega-newton is used. Kilo-newton is represented by kN.

One kilo-newton = $10^3$ newton

or 1 kN = $10^3$ N

and One mega newton = $10^6$ Newton

The large quantities are represented by kilo, mega, giga and terra. They stand for:

Kilo = $10^3$ and represented by ……k

Mega = $10^6$ and represented by

Giga = $10^9$ and represented by …….G

Tera = $10^{12}$ and represented by ……..T

Thus mega newton means 106 newton and is represented by MN. Similarly, giga newton means $10^9$ N and is represented by GN. The symbol TN stands for $10^{12}$ N. The small quantities are represented by milli, micro, nano and pico. They are equal to

Milli = $10^{-3}$ and represented by …….m

Micro = $10^{-6}$ and represented by …….μ

Nano = $10^{-9}$ and represented by ……..n

Pico = $10^{-12}$ and represented by ……..p.

Thus milli newton means $10^{-3}$ newton and is represented by mN. Micro newton means $10^{-6}$ N and is represented by μN.

Weight of a body is the force with which the body is attracted towards earth. If W = weight of a body, $m$ = mass in kg, then W = $m \times g$ Newtons

If mass, $m$ of the body is 1 kg, then its weight will be,

$$W = 1\ (\text{kg}) \times 9.81\frac{\text{m}}{\text{s}^2} = 9.81\ \text{N}. \qquad \left(\because \text{N} = \text{kg}\ \frac{\text{m}}{\text{s}^2}\right)$$

**Problem 1.1.** Two forces of magnitude 10 N and 8 N are acting at a point. If the angle between the two forces is 60°, determine the magnitude of the resultant force.

**Sol.** Given :

Force P = 10 N

Force Q = 8 N

Angle between the two forces, $\alpha$ = 60°

The magnitude of the resultant force (R) is given by equation (1.1)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = R = \sqrt{10^2 + 8^2 + 2 \times 10 \times 8 \times \cos 60°}$$

$$= \sqrt{100 + 64 + 2 \times 10 \times 8 \times \frac{1}{2}} \qquad \left(\because \cos 60° = \frac{1}{2}\right)$$

$$= \sqrt{100 + 64 + 80} = \sqrt{244} = \mathbf{15.62\ N.\ Ans.}$$

**Problem 1.2.** Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $20 \times \sqrt{3}$ N, find magnitude of each force.

**Sol.** Given : Angle between the force, $\alpha = 60°$

Resultant, $R = 20 \times \sqrt{3}$

The forces are equal. Let P is the magnitude of each force.

Using equation (1.3), we have

$$R = 2P \cos \frac{\alpha}{2} \quad \text{or} \quad 20 \times \sqrt{3} = 2P \times \cos \frac{60°}{2} = 2P \cos 30°$$

$$= 2P \times \frac{\sqrt{3}}{2} = P \times \sqrt{3} \qquad (\because \cos 30° = \frac{\sqrt{3}}{2})$$

$$\therefore \quad P = \frac{20 \times \sqrt{3}}{\sqrt{3}} \text{ 20 N.}$$

$\therefore$ Magnitude of each force = **20 N. Ans.**

**Problem 1.3.** The resultant of the two forces, when they act at an angle of 60° is 14 N. If the same forces are acting at right angles, their resultant is $\sqrt{136}$ N. Determine the magnitude of the two forces.

**Sol.** Given : **Case I**

Resultant, $R_1 = 14$ N

Angle, $\alpha = 60°$

**Case II**

Resultant, $R_2 = \sqrt{136}$ N.

Angle, $\alpha = 60°$

Let the magnitude of the two forces are P and Q.

Using equation (1.1) for case I.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

or $$14 = \sqrt{P^2 + Q^2 + 2PQ \times \cos 60°} = \sqrt{P^2 + Q^2 + 2PQ \times \frac{1}{2}}$$

or $$14 = \sqrt{P^2 + Q^2 + PQ}$$

Squaring $$196 = P^2 + Q^2 + PQ \qquad \text{(i)}$$

Using equation [1.2 (A)] for class II,

$$R = \sqrt{P^2 + Q^2} \quad \text{or} \quad \sqrt{136} = \sqrt{P^2 + Q^2}$$

or $\quad 136 = P^2 + Q^2$ (Squaring both sides) ...(ii)

Subtracting equation (ii) from equation (i), we get

$$196 - 136 = P^2 + Q^2 + PQ - (P^2 + Q^2)$$

$$600 = PQ \quad \text{...(iii)}$$

or Multiplying the above equation by two, we get $120 = 2\,PQ$ ...(iv)

Adding equation (iv) to equation (ii), we get $136 + 120 = P^2 + Q^2 + 2PQ$

or $\quad 256 = P^2 + Q^2 + 2PQ$ or $(16)^2 = (P + Q)^2$

or $\quad 16 = P + Q$

$\therefore \quad P = (16 - Q)$ ...(v)

Substituting the value of P in equation (iii), we get

$$60 = (16 - Q) \times Q = 16Q - Q^2 \text{ or } Q^2 - 16Q + 60 = 0$$

∴ This is a quadratic equation.

$$\therefore \quad Q = \frac{16 \pm \sqrt{(-16)^2 - 4 \times 60}}{2} = \frac{16 \pm \sqrt{256 - 240}}{2} = \frac{16 \pm 4}{4}$$

$$= \frac{16+4}{4} \quad \text{and} \quad \frac{16-4}{2} = 10 \text{ and } 6.$$

Substituting the value of Q in equation (*v*), we get

$$P = (16 - 10) \text{ or } (16 - 6) = 6 \text{ or } 10.$$

∴ Hence the two forces are **10 N** and **6 N. Ans.**

**Problem 1.4.** The resultant of two concurrent forces is 1500 N and the angle between the forces is 90°. The resultant makes an angle of 36° with one of the force. Find the magnitude of each force.

**Sol.** Given :

Resultant, R = 1500 N

Angle between the forces, $\alpha$ = 90°

Angle made by resultant with one force, θ = 36°

Let P and Q are two forces.

Using equation (1.2), $\tan\theta = \dfrac{Q\sin\alpha}{P+Q\cos\alpha}$

or $\quad \tan 36° = \dfrac{Q\sin\alpha}{P+Q\cos\alpha} = \dfrac{Q \times 1}{P+Q \times 0} = \dfrac{Q}{P}$ or $0.726 = \dfrac{Q}{P}$

$$Q = 0.726\,P \quad \text{...(i)}$$

Using equation (1.1), $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

or $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

or $1500^2 = P^2 + (0.726P)^2 + 2P(0.726P) \times \cos 90°$

$(\because Q = 0.726P)$

or $1500^2 = P^2 + 0.726P^2 + 0$ $(\because \cos 90° = 0)$

$= 1.527\ P^2$

$\therefore$ $P = \sqrt{\dfrac{1500^2}{1.527}} = \dfrac{1500}{1.2357} = 1213.86$ N

Substituting the value of P in equation (i), we get

$Q = 0.726 \times 1213.86 =$ **881.26 N. Ans,**

**Alternate Method.** Refer to Fig. 1.7. Consider triangle OAC.

Using sine rule, we get

$$\frac{\sin 90°}{R} = \frac{\sin 36°}{Q} = \frac{\sin 54°}{P}$$

or $$\frac{\sin 90°}{R} = \frac{\sin 36°}{Q}$$

or $$Q = \frac{R\sin 36°}{\sin 90°} \text{ (where R = 1500 N)}$$

$$= \frac{1500 \times 0.5877}{1} = \textbf{881.67 N. Ans.}$$

Also, we have $$\frac{\sin 90°}{R} = \frac{\sin 54°}{P}$$

$\therefore$ $$P = \frac{R\sin 3°}{\sin 90°} = \frac{1500 \times 0.8090}{1}$$

$= $ **1213.52 N. Ans.**

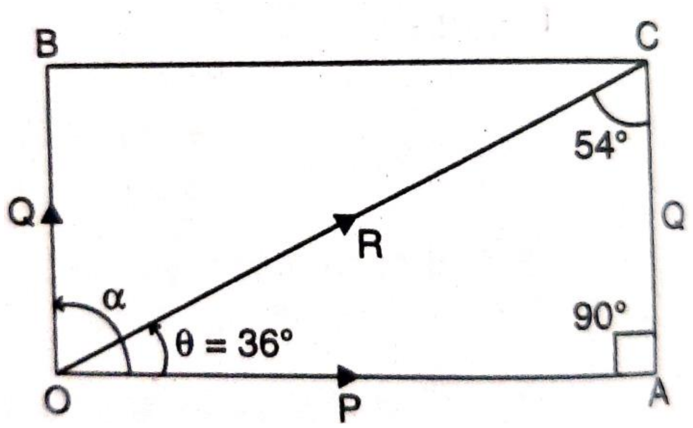



**Fig. 1.7**

**Problem 1.5.** The sum of two concurrent forces P and Q is 270 N and their resultant is 180 N. The angle between the force P and resultant R is 90°. Find the magnitude of each force and angle between them.

**Sol.** Given :

Sum of two concurrent forces = 270 N or P + Q = 270 N

Resultant, R = 180 N

Angle between force P and resultant R = 90°

This means = 90°

Find : (i) Magnitude of P and Q

(ii) Angle between P and Q (i.e., angle $\alpha$)

Using equation (1.2), $\tan\theta = \dfrac{Q\sin\alpha}{P+Q\cos\alpha}$

or $$\tan 90° = \frac{Q\sin\alpha}{P+Q\cos\alpha}$$

But tan 90° = ∝ (i.e., infinity). This is only possible when $P + Q\cos\alpha = 0$

$$\therefore \quad P = -Q\cos\alpha \quad \quad \ldots(i)$$

The above result can also be obtained by using alternate method.

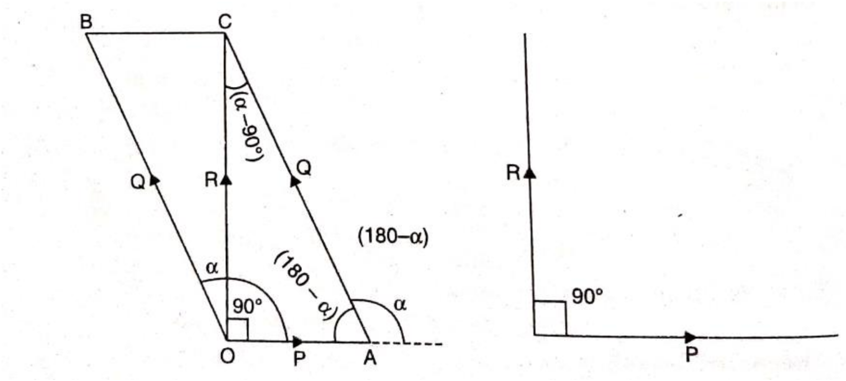



**Fig. 1.8**

**Alternate Method.** Refer to Fig. 1.8. Consider triangle OAC in which θ = 90°, ∠OAC = 180 – $\alpha$, $\angle ACO = \alpha - \theta = \alpha - 90°$.

Using sine rule, we get $$\frac{\sin 90°}{Q} = \frac{\sin(180° - \alpha)}{R} = \frac{\sin(\alpha - 90°)}{P}$$

From first and last terms, we get $$\frac{\sin 90°}{Q} = \frac{\sin(\alpha - 90°)}{P}$$

or $$\frac{1}{Q} = \frac{\cos\alpha}{P}$$

$[\therefore \sin(\alpha - 90) = \sin[-(90 - \alpha)] = -\sin(90 - \alpha) = -\cos\alpha]$

$\therefore \quad P = -Q\cos\alpha$

This is the same result as given by equation (i) above.

Using equation (1.1), $\quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

Squaring to both sides, we get $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$$= P^2 + Q^2 + 2P(-P)$$

$(\therefore$ From equation (i), $Q\cos\alpha = -P)$

$$= P^2 + Q^2 + 2P^2 = Q^2 - P^2 = (Q + P)(Q - P)$$

or $\quad 180^2 = 270\,(Q - P) \quad (\therefore R = 180, Q + P = 270)$

or $\quad 32400 = 270\,(Q - P)$

$\therefore \quad Q - P = \frac{32400}{270} = 120$

But $\quad P + Q = 270 \quad$ (given)

Addition the above two equation, we get $2Q = 270 + 120 = 390$

$\therefore \quad$ **Q = 195 N. Ans.**

and $\quad P = 270 - Q = 270 - 195 =$ **75 N. Ans.**

**Value of angle** $\alpha$

Substituting the value of P and Q in equation (i)

$P = -Q\cos\alpha \quad$ or $\quad 75 = -195\cos\alpha$

or $\quad \cos\alpha = \frac{-75}{195} = -0.3846$

$\therefore \quad \alpha = \cos^{1}(-0.3846) =$ **112.618° Ans.**

**Problem 1.6.** A weight of 1000 N is supported by two chains as shown in Fig. 1.9. Determine the tension in each chain.

**Sol.** Given: Weight at C = 1000 N

$\angle CAB = 30°$

$\angle CBA = 60°$

$\angle ACB = 90°$

In right angled triangle ADC

$\angle CAB = 90° - 30° = 60°$

In right angled triangle BDC

$$\angle BCD = 90° - 60° = 30°$$

$\therefore$ $\angle ACE = 180° - 60° = 120°$

$$\angle BCE = 180° - 30° = 150°$$

Let $T_1$ = Tension in chain No. 1

$T_2$ = Tension in chain No. 2.

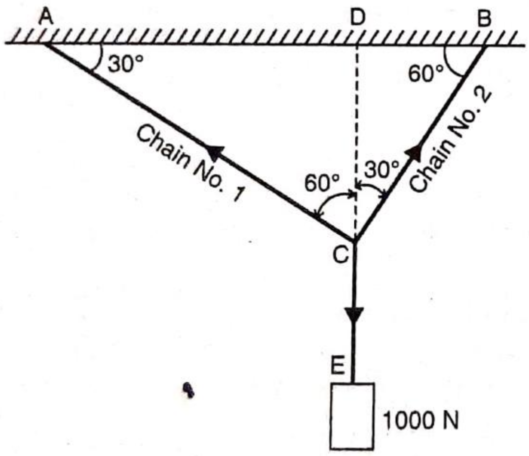



**Fig. 1.9**

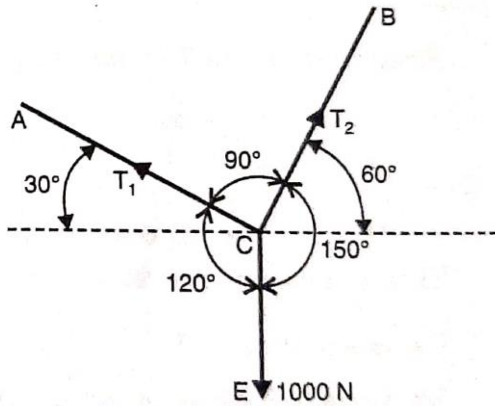



**Fig. 1.10**

Applying Lami's theorem at point C (Refers Fig. 1.9 (a)).

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{T_2}{\sin 90°}$$

or $$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = 1000 \qquad (\because \sin 90° = 1)$$

$\therefore$ $T_1 = 1000 \sin 150° = 1000 \times .5 =$ **500 N. Ans.**

and $T_2 = 1000 \sin 120° = 1000 \times .866 =$ **866 N. Ans.**

## 1.7 RESOLUTION OF A FORCE

Resolution of a force means "finding the components of a given force in two given directions."

Let a given force be R which makes an angle θ with X-axis as shown in Fig. 1.10. It is required to find the components of the force R along X-axis and Y-axis.

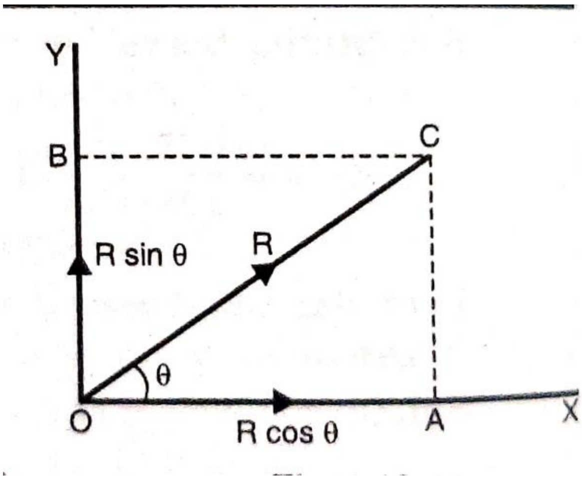



**Fig. 1.10**

Components of R along X-axis = R cos θ.

Components of R along Y-axis = R sin θ.

Hence, the resolution of forces is the process of finding components of forces in specified directions.

**1.7.1. Resolution of a Number of Coplanar Forces.** Let a number of coplanar forces (forces acting in one plane are called coplanar forces) $R_1$, $R_2$, $R_3$,...... are acting at a point as shown in Fig. 1.11.

Let $\theta_1$ = Angle made by $R_1$ with X-axis

$\theta_2$ = Angle made by $R_2$ with X-axis

$\theta_3$ = Angle made by $R_3$ with X-axis

H = Resultant component of all forces along X-axis

V = Resultant component of all forces along Y-axis

R = Resultant of all forces

θ = Angle made by resultant with X-axis.

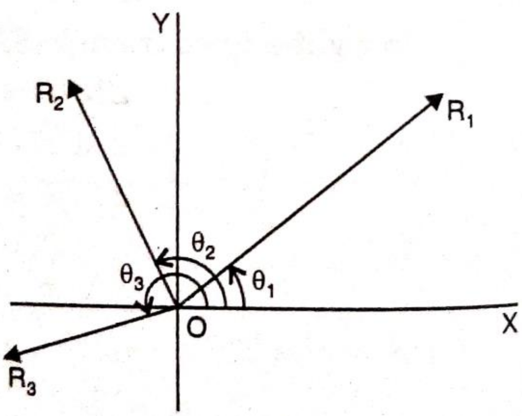



**Fig. 1.11**

Each force can be resolved into two components, one along X-axis and other along Y-axis.

Component of $R_1$ along X-axis = $R_1 \cos \theta_1$

Component of $R_1$ along Y-axis = $R_1 \sin \theta_1$

Similarly, the components of $R_2$ and $R_3$ along X-axis and Y-axis are $(R_1 \cos \theta_2, R_2 \sin \theta_2)$ and $(R_3 \sin \theta_3, R_3 \sin \theta_3)$ respectively.

Resultant components along X-axis

= Sum of components of all forces along X-axis.

$\therefore \quad H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3 + \ldots \qquad \ldots(1.6)$

Resultant component along Y-axis.

= Sum of components of all forces along Y-axis.

$\therefore \quad V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3 + \ldots \qquad \ldots(1.7)$

Then resultant of all the forces, $R = \sqrt{H^2 + V^2} \qquad \ldots(1.8)$

The angle made by R with X-axis is given by, $\tan \theta = \frac{V}{H} \qquad \ldots(1.9)$

**Problem 1.7.** Two forces are acting at a point O as shown in Fig. 1.12. Determine the resultant in magnitude and direction.

**Sol.** The above problem has been solved earlier.

Hence it will be solved by resolution of forces.

Force $\quad P = 50$ N and force $Q = 100$ N.

Let us first find the angles made by each force with X-axis.

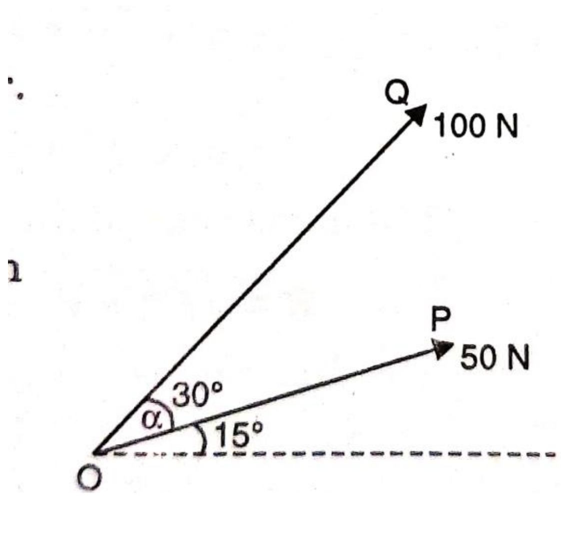



**Fig. 1.12**

Angle made by P with x-axis = 15°

Angle made by Q with x-axis = 15 + 30 = 45°

Let H = Sum of components of all forces along X-axis.

V = Sum of components of all forces along Y-axis.

The sum of components of all forces along X-axis is given by,

$$H = P \cos 15° + Q \cos 45°$$

$$= 50 \times \cos 15° + 100 \cos 45° = 119 \text{ N}$$

The sum of components of all forces along Y-axis is given by,

$$V = P \sin 15° + Q \sin 45°$$

$$= 50 \sin 15° + 100 \sin 45° = 83.64 \text{ N}$$

The magnitude of the resultant force is given by equation (1.8),

$$R = \sqrt{H^2 + V^2} = \sqrt{199^2 + 83.64^2} = \mathbf{145.46\ N.\ Ans.}$$

The direction of the resultant force is given by equation (1.9), $\tan\theta = \frac{V}{H} = \frac{83.64}{119}$.

Here θ is the angle made by resultant R with *x*-axis.

**Problem 1.8.** Three forces of magnitude 40 kN, 15 kN and 20 kN are acting at a point O as shown in Fig. 1.13. The angles made by 40 kN, 15 kN and 20 kN forces with X-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.

**Sol.** Given:

$R_1 = 40$ kN, $\theta_1 = 60°$

$R_2 = 15$ kN, $\theta_2 = 120°$

$R_3 = 20$ kN, $\theta_3 = 240°$

The sum of components of all forces along X-axis is given by equation (1.6) as

$$H = R_1 \cos\theta_1 + R_2 \cos\theta_2 + R_3 \cos\theta_3$$

$$= 40 \times \cos 60° + 15 \times \cos 120° + 20 \times \cos 240°$$

$$= 40 \times \frac{1}{2} + 15 \times \left(-\frac{1}{2}\right) + 20 \times \left(-\frac{1}{2}\right)$$

$$= 20 - 7.5 - 10 = 2.5 \text{ kN.}$$

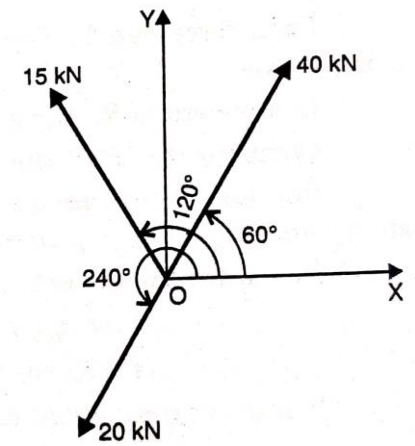



**Fig. 1.13**

The resultant component along Y-axis is given by equation (1.7) as

$$V = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3$$

$$= 40 \times \sin (60°) + 15 \times \sin (120°) + 20 \times \sin (240°)$$

$$= 40 \times \frac{\sqrt{3}}{3} + 15 \times \frac{\sqrt{3}}{2} + 20 \times \left(\frac{-\sqrt{3}}{2}\right)$$

$$= 20 \times \sqrt{3} + 7.5 \times \sqrt{3} - 10 \times \sqrt{3} = 17.5 \times \sqrt{3} \text{ kN} = 30.31 \text{ kN.}$$

The magnitude of the resultant force is given by equation (1.8)

$$R = \sqrt{H^2 + V^2} = \sqrt{2.5^2 + 30.31^2} = \mathbf{30.41\ N.\ Ans.}$$

The direction of the resultant force is given by equation (1.9)

$$\tan \theta = \frac{V}{H} = \frac{30.31}{2.5}.= 12.124 = \tan 85.28°$$

$$\therefore \qquad \theta = \mathbf{85.28° \quad or \quad 85° \ 16.8'.\ Ans.}$$

**Problem 1.9** four forces of magnitude 10 kN, 15 kN, 20 kN and 40 kN are acting at a point O as shown in Fig. 1.14. The angles made by 10 kN, 15 kN, 20 kN and 40 kN with X-axis are 30°, 60°, 90° and 120° respectively. Find the magnitude and direction of the resultant force.

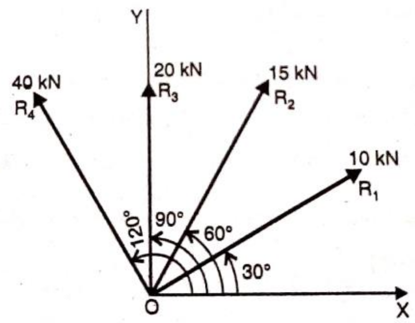



**Fig. 1.14**

**Sol.** Given :

$R_1 = 10$ kN and $\theta_1 = 30°$,

$R_2 = 15$ kN and $\theta_2 = 60°$,

$R_3 = 20$ kN and $\theta_3 = 90°$,

$R_4 = 25$ kN and $\theta_4 = 120°$,

The resultant components along X-axis is given by equation (1.6) as

$H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3, + R_4 \cos \theta_4$

$= 10 \times \cos 30° + 15 \cos 60° + 20 \cos 90° + 40 \cos 120°$

$= 10 \times \frac{\sqrt{3}}{2} + 15 \times \frac{1}{2} + 20 \times 0 + 40 \times \left(-\frac{1}{2}\right)$

[$\because \cos 90° = 0$ and $\cos 120° = -\frac{1}{2}$]

Negative sign means that *H* is acting along *OX'* as shown in Fig. 1.15.

The resultant component along Y-axis is given by equation (1.7) as

$V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3,$

$R_4 \sin \theta_4$

$= 10 \sin 30° + 15 \sin 60° + 20 \sin 90° + 40 \sin 120°$

$= 10 \times \frac{1}{2} + 15 \times \frac{\sqrt{3}}{2} + 20 \times 1 + 40 \times \frac{\sqrt{3}}{2}$

$= 5 + 7.5 \times \sqrt{3} + 20 + 20 \times \sqrt{3}$

$= 25 + 27.5 \times \sqrt{3} = 72.63$ kN.

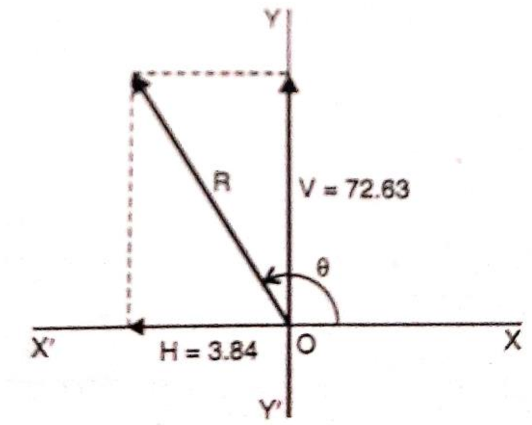



**Fig. 1.15**

Positive sign means that *V* is acting along OY as shown in Fig. 1.15.

The magnitude of the resultant force is given by equation (1.8) as

$R = \sqrt{H^2 + V^2} = \sqrt{(-3.84)^2 + 72.63^2}$

$= \sqrt{14.745 + 5275.117} =$ **72.73 kN. Ans.**

The direction of the resultant force is given by equation (1.9) as

$\tan \theta = \frac{V}{H} = \frac{72.63}{-3.84} = -18.91$

From Fig. 1.15 is clear that θ lies 90° and 180°..

The angle whose tangent is 18.91° is 86.97.

$\therefore \theta = (180° - 86.97°) =$ **93.03°, Ans.**

## 1.8 LAWS OF MECHANICS

The following basic laws and principles are considered to be the foundation of mechanics :

(i) Newton's first and second laws of motion
(ii) Newton's third law
(iii) The gravitational law of attraction
(iv) The parallelogram law
(v) The Principle of Transmissibility of forces.

**1.8.1. Newton's First and Second Laws of Motion.** Newton's first law states, "Every body continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it."

Newton's second law states, "The net external force acting on a body in a direction is directly proportional to the rate of change of momentum in that direction."

**1.8.2, Newton's Third Law.** Newton's third law states, "To every action there is always equal and opposite reaction."

Fig. 1.16 shows two bodies A and B which are placed one above the other on a horizontal surface.

Here $F_1$ = Force exerted by horizontal surface on body A (action)

$-F_1$ = Force exerted by body A on horizontal surface (reaction)

$F_2$ = Force exerted by body A on body B (action)

$-F_2$ = Force exerted by body B on body A (reaction)

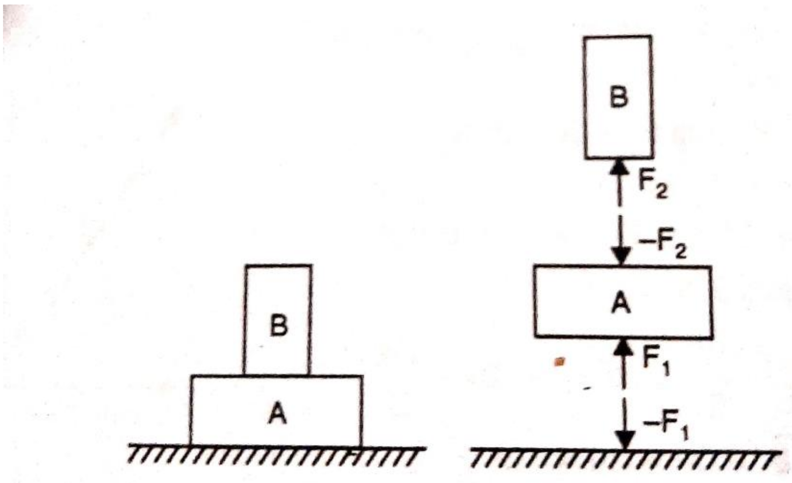



**Fig. 1.16**

**1.8.3. The Gravitational Law of Attraction.** It states that two bodies will be attracted towards each other along their connecting line with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Refer to Fig. 1.17.

Let $m_1$ = Mass of first body

$m_2$ = Mass of second body

$r$ = Distance between the centre of bodies

F = Force of attraction between the bodies.

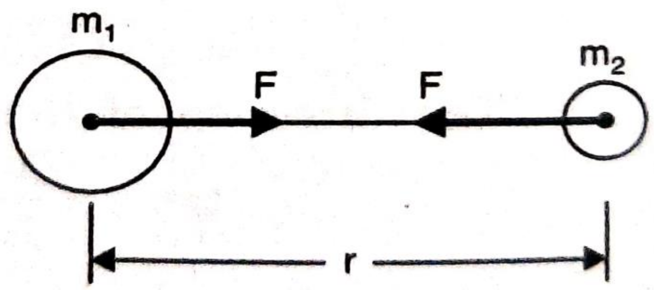



**Fig. 1.17**

Then according to the law of gravitational attraction

$$F \propto m_1 \,.\, m_2$$

$$\propto \frac{1}{r_2}$$

or $$F \propto \frac{m_1 m_2}{r_2}$$

or $$F = G \; \frac{m_1 m_2}{r_2}$$

where G = Universal gravitational constant of proportionality.

**1.8.4. The Parallelogram Law.** This law has been already defined. It states that if two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

**1.8.5. The Principle of Transmissibility of Forces**. It states that if a force, acting at *a* point on a rigid *body, is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

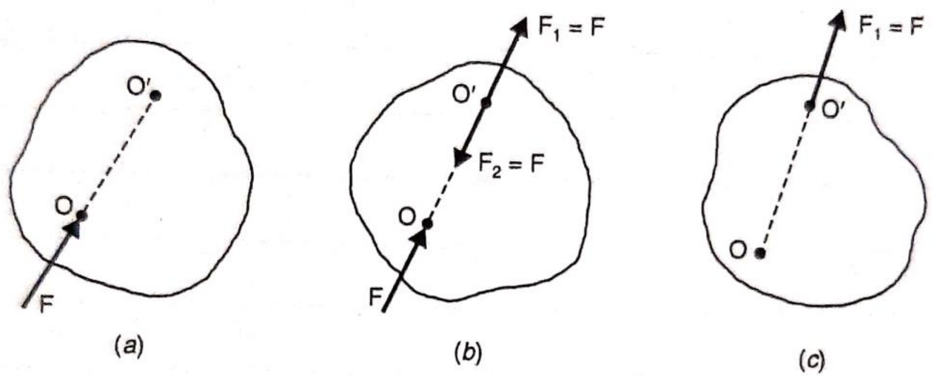



**Fig. 1.18**

When two forces are equal, opposite and collinear they are in equilibrium and their resultant is zero. The combined effect of these two forces on a rigid body is equivalent to that of no force at all.

Now consider a forco F acting at point on a rigid body as shown in Fig. 1.18 (a). On this rigid body, "there is another point O' in the line of action of the force F. Suppose at this point O', two equal and opposite forces F, and F, (each equal to F and collinear with F) are applied as shown in Fig. 1.18(b). The combined effect of these two forces on the body is equivalent to that of no force at all. Now consider force F and $F_2$. The force Find F and $F_2$ being equal and opposite, will cancell each other, leaving a force $F_1$ at point O' as shown in Fig. 1.18 (c). But force $F_1$ is equal to force F.

The original force F acting at point O, has been transferred to point O' which is along the line of action of F without changing the effect of the force on the rigid body. Hence any force acting at a point on a rigid body can be transmitted to act at any other point along its line of action without changing its effect on the rigid body. This proves the principle of transmissibility of a force.

## STUDENT ACTIVITY

**1.** What do you mean by rigid body?

**2.** What do you mean by scalar and vector quantities?

## SUMMARY

1. Engineering mechanics is divided into statics and dynamics. The study of a body at rest is known as statics whereas the study of a body in motion is known as dynamics.
2. A quantity which is completely specified by magnitude and direction is known as vector quantity.
3. A particle is a body of infinitely small volume and is considered to be concentrated at a point.
4. Law of parallelogram of forces states that "If two forces, acting at a point be
   represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.
5. If two forces P and Q act at a point and the angle between the two forces be α, then the resultant is given by
   $$R = \sqrt{P^2 + Q^2 + 2PQ\cos}$$
   and the angle made by the resultant with the direction of force P is expressed as
   $$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P+Q\cos\alpha}$$
6. If the two forces P and Q are equal and are acting at an angle a between them, then the resultant is given by
   $$R = 2P\cos\frac{\alpha}{2}$$
   and angle made by the resultant is expressed as $\theta = \frac{\alpha}{2}$.
7. According to Lami's theorem, "If three forces acting at a point are equilibrium, each force will be proportional to the sine of the angle between the other two forces."
8. The relation between newton and dyne is given by One newton = $10^5$ dyne.
9. Gravitational law of attraction is given by,
   $$F = G\ \frac{m_1 m_2}{r_2}$$
   where G = Universal gravitational constant
   $m_1, m_2$ = Mass of bodies
   $r$ = Distance between the bodies
   F = Force of attraction between the bodies.

## TEST YOURSELF

**(A) Theoretical Problems**

1. Define and explain the terme : Mechanics and applied mechanics,
2. Describe in details the importance and necessity of applied mechanics.

3. What do you mean by rigid body?
4. State the different branches of mechanics.
5. What do you mean by scalar and vector quantities?
6. Define the law of parallelogram of forces. What is the use of this law?
7. State triangle law of forces and Lami'o theorem.
8. Two force P and Q are acting at a point in a plane. The angle between the forces is '$\alpha$'. Prove that the resultant (R) of the two forces is given by R = $\sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$
9. Prove that one newton is equal to $10^5$ dyne.
10. Write the SI units of : Force, moment and velocity.
11. What do you mean by resolution of a force ?
12. A number of coplanar forces are acting at a point making different angles with $x$-axis. Find an expression for the resultant force. Find also the angle made by the resultant force with $x$-axis.
13. State and explain the pinciple of transmissibility of forces.

**(B) Numerical Problems**

1. Determine the magnitude of the resultant of the two forces of magnitude 12 N and 9 N acting at a point if the angle between the two forces is 30°,
[**Ans.** 20.3 N]
2. Find the magnitude of two equal forces acting at a point with an angle of 60° between them, if the resultant is equal to 30 × $\sqrt{3}$ N.
[**Ans.** 30 N]
3. The resultant of two forces when they act at right angles is 10 N, whereas when they act at an angle of 60° the resultants is $\sqrt{148}$. Determine the magnitude of the two forces.
[**Ans.** 8 N and 6 N]
4. Three forces of magnitude 30 kN, 10 kN and 15 kN are acting at a point O. The angles made by 30 kN force, 10 kN force and 15 kN force with $x$-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.
[**Ans.** 21.79 kN, 83° 24]
5. A weight of 800 N is supported by two chains as shown in Fig. 1.19. Determine the tension in each chain.
[**Ans.** 273.5 N, 751.7 N]

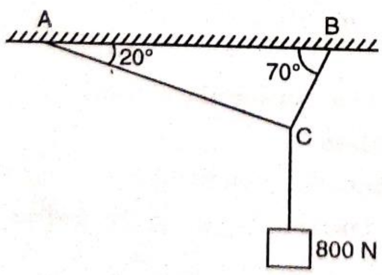



**Fig. 1.19**

6. Two forces magnitude 15 N and 12 N are acting at a point. If the angle between the two forces is 60°, determines the resultant of the forces in magnitude and direction.

   [**Ans.** 23.43 N, 26.3°]

7. Four forces of magnitude P, 2P, $3 \times \sqrt{3}$ P and 4P are acting at a point O. The angles may be these forces with $x$-axis are 0°, 60°, 150° and 300°, respectively. Find the magnitude and direction of the resultant force.

   [**Ans.** P, 1200]

# 2 LAW OF FORCES

## LEARNING OBJECTIVES

- Force
- Unit of Force
- Bow's Notation
- Coplanar and Space Force System
- Resultant of Several Forces
- Resultant of Coplanar Forces
- Resultant of Collinear Coplanar Forces
- Resultant of Concurrent Coplanar Forces
- Action and Reaction
- Free Body Diagram

## 2.1 FORCE

Force is defined as any action that tends to change the state of rest of a body to which it is applied. Different kind of forces are :

(i) Gravity force
(ii) Simple pull or push exerted on a body by hand
(iii) Force due to the pressure of steam or gas on the piston of a cylinder
(iv) Force due to frictional resistance between contact surfaces.

The force can be specified by :

(i) Its magnitude
(ii) The point of application and
(iii) Its direction

Any quantity that possesses direction as well as magnitude is known as a vector quantity. Hence force is a vector quantity.

## 2.2 UNIT OF FORCE

Unit of force is a Newton. It is represented by N. Newton is a force which acts on a mass of 1 kilogram and produces an acceleration of $1 m/s^2$. Hence

$$1N=(1kg)\times(1m/s^2)=1$$

## 2.3 BOW'S NOTATION

According to Bow's notation, a force is represented by two Capital letters which are written on either side of the line of action of the force. A force with letters A and B on either side of the line of action is shown in Fig. 2.1. This force will be called AB. The magnitude of the force is represented by the length of the line. Direction of the force is given by the arrow as shown in Fig. 2.1.

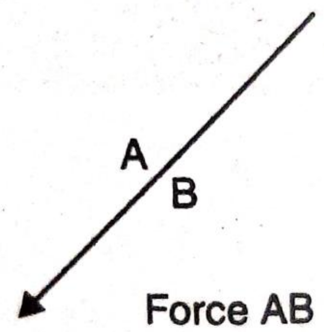



**Fig. 2.1**

## 2.4 COPLANAR AND SPACE FORCE SYSTEM

Coplanar forces means the forces in a plan. The word collinear stands for the forces which are having common lines of action whereas the word concurrent stands for the forces which intersect at a common point. When several forces act on a body, then they are called a force system or a system of force. In a system in which all the force lie in the same plane, it is known as coplanar force system.

A force system may be coplanar or non-coplanar. If in a system all the forces lie in the same plane then the force system is known as coplanar. But if in a system all the forces lie in a different planes, then the force system is known as non-coplanar. Hence a force system is classified as shown in Fig. 2.1 (a).

**Force System**

- 2. Coplanar
  - Collinear
  - Concurrent
  - Parallel
  - Non-concurrent non-parallel
- 1. Non-Coplanar
  - Concurrent
  - Parallel
  - Non-co non-p

**2.4.1 Coplanar Collinear.** Fig.2.2 shows three forces $F_1$, $F_2$ and $F_3$ acting in a plane. These three forces are in the same line i.e., these three forces are having a common line of action. This system of forces are known as coplanar collinear force system. Hence in coplanar collinear system of forces, all the forces act in the same plane and have a common line of action.

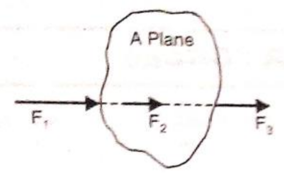



**Fig. 2.1** Coplanar Collinear Forces.

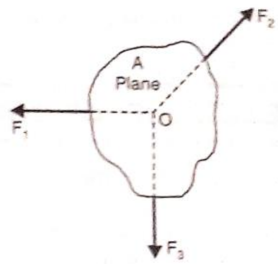



**Fig. 2.3** Concurrent Coplanar Forces

**2.42 Coplanar Concurrent.** Fig. 2.3 shows three forces $F_1$, $F_2$ and $F_3$ acting in a plane and these forces intersect or meet at a common point O. This system of forces is known as coplanar concurrent force system. Hence in coplanar concurrent system of forces, all the forces act in the same plane and they intersect at a common point.

**2.43 Coplanar Parallel.** Fig.2.4 shows three forces $F_1$, $F_2$ and $F_3$ acting in a plane and these forces are parallel. This system of forces is known as coplanar parallel force system. Hence in coplanar parallel system of forces, all the forces act in the same plane and are parallel.

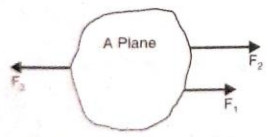



**Fig. 2.4.** Coplanar Parallel Forces.

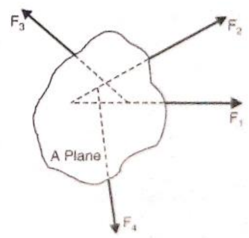



**Fig. 2.5** Non-Concurrent Non-Parallel

**2.4.4. Coplanar Non-concurrent Non-parallel.** Fig.2.5 shows four forces $F_1$, $F_2$ and $F_3$ acting in a plane. The lines of action of these forces lie in the same plane but they are neither parallel nor meet or intersect at a common point. This system of forces is known as coplanar non-concurrent non-parallel force system. Hence in coplanar non- concurrent non-parallel system of forces act in the same plane but the forces are neither parallel nor meet at a common point. This force system is also known as general system of force.

## 2.5 RESULTANT OF SEVERAL FORCES

When a number of coplanar forces are acting on a rigid* body, then these forces can be replaced by a single force which has the same effect on the rigid body as that of all the forces acting together, then this single force is known as the resultant of several forces. Hence a single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body, is known as the resultant force.

## 2.6 RESULTANT OF COPLANAR FORCES

The resultant of coplanar forces may be determined by the following two methods:

1. Graphical method.
2. Analytical method.

The resultant of the following coplanar forces will be determined by the above two methods:

1. Resultant of collinear coplanar forces.
2. Resultant of concurrent coplanar forces.

## 2.7 RESULTANT OF COLLINEAR COPLANAR

As defined in Art.2.4.1, collinear coplanar forces are those forces which act in the same plane and have a common line of action. The resultant of those forces are obtained by Analytical method or Graphical method.

**2.7.1 Analytical Method.** The resultant is obtained by adding all the forces if they are acting in the same direction. If any one of the forces is acting in the opposite direction, then resultant is obtained by subtracting that force.

Fig.2.6 shows three collinear coplanar forces $F_1$, $F_2$ and $F_3$ acting in on a rigid body in the same direction. Their resultant R will be sum of these forces.

$$R = F_1 + F_2 + F_3 \qquad \ldots(2.1)$$

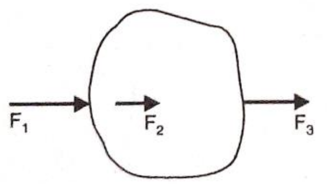



**Fig 2.6**

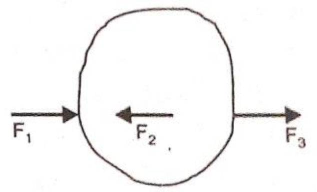



**Fig. 2.7**

If any one of these forces (say force $F_2$) is acting in the opposite direction, as shown in Fig.2.7, then their resultant will be given by

$$R = F_1 + F_2 + F_3 \qquad \ldots(2.2)$$

**2.7.2 Graphical Method.** Some suitable scale is chosen and vectors re drawn to the chosen scale. These vectors are added/or subtracted to find the resultant. The resultant of the three collinear forces $F_1$, $F_2$ and $F_3$ acting in the same direction will be obtained by adding all the vectors. In Fig.2.8, the force $F_1$ = ab to some force $F_2$ = bc and force $F_3$ = cd. Then the length and represents the magnitude of the resultant on the scale chosen.

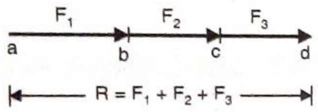



**Fig 2.8**

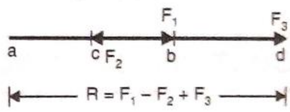



**Fig. 2.9**

The resultant of the forces $F_1$, $F_2$ and $F_3$ acting on a body shown in Fig.2.7 will be obtained by subtracting the vector $F_3$. This resultant is shown in Fig.2.9 in which the force $F_1$ = *ab* to some suitable scale. This force is force is acting from a to b. The force $F_2$ is taken equal to *bc* on the same scale in opposite direction. This force is acting from b to c. The force $F_3$ is taken equal to cd. This force is acting from c to d. The resultant force is represented in magnitude by ad on the chosen scale.

**Problem 2.1.** Three collinear horizontal forces of magnitude 200N, 100N and 300 N are acting on a rigid body. Determine the resultant of the forces analytically and graphically when

(i) All the forces are acting in the same direction.

(ii) The force 100 N acts in the opposite direction.

**Sol.** Given: $F_1$ = 200 N, $F_2$ = 100 N, and $F_3$ = 300 N

**(a) Analytical method**

(i) When all the forces are acting in the same direction, the resultant is given by equation (2.1) as

$$R = F_1 + F_2 + F_3 + F_4 = 200 + 100 + 300 = \mathbf{600\ N.\ Ans.}$$

(ii) When the force 100 N acts in the opposite direction, then resultant sin given by equation (2.2) as

$R = F_1 + F_2 + F_3 + F_4 = 200 - 100 + 300 =$ **400 N. Ans.**

**(b) Graphical method.** Select a suitable scale. Suppose 100 N = 1 cm. Then to this scale, we have

$F_1 = \frac{200}{100} = 2$ cm, $F_2 = \frac{100}{100} = 1$ cm

and $F_3 = \frac{300}{100} = 3$ cm

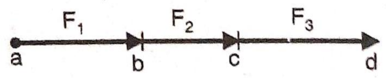



**Fig. 2.10**

(i) When all the forces act in the same direction.

Draw vectors = *ab* = 2 cm to represent $F_1$

vectors = *bc* = 1 cm to represent $F_2$ and

vectors = *cd* = 3 cm to represent $F_3$ as shown in Fig. 2.10.

Measurement vector *ad* which represents the resultant.

By measurement, length *ad* = 6 cm

$\therefore$ resultant = length *ad* × chosen scale

($\therefore$ Chosen scale is 1 cm = 100 N)

= 6 × 100 = **600 N. Ans**

(ii) When force 100 N = $F_2$ acts in the opposite direction

Draw length *ab* = 2 cm to represent force $F_1$

From b, draw bc= 1 cm in the opposite direction to represent $F_2$. From c, draw cd= 3 cm to represent $F_3$ as shown in Fig. 2.10(a).

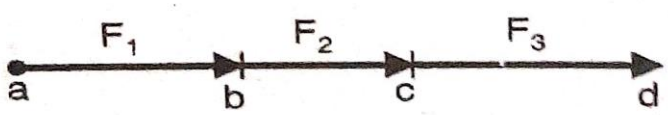



**Fig 2.10**

Measure length ad. This gives the resultant.
By measurement, length ad = 4 cm
Resultant = Length ad × chosen scale
= 4 × 100 = **400 N. Ans.**

## 2.8 RESULTANT OF CONCURRENT COPLANAR

As defined in Art. 2.4.2, concurrent coplanar forces are those forces which act in the same plane and they intersect or meet at a common point. We will consider following two cases :

(i) When two forces act at a point.
(ii) When more than two forces act at a point.

### 2.8.1, When Two Forces Act at a Point

**(a) Analytical method.** In Art. 1.5.4, we have mentioned that when two forces act at a point, their resultant is found by the law of parallelogram of forces. The magnitude of resultant is obtained from equation (1.1) and the direction of resultant with one of the forces is obtained from equation (1.2).

Suppose two forces P and Q act at point O as shown in Fig. 2.11 and $\alpha$ is the angle between them. Let $\theta$ is the angle made by the resultant R with the direction of force P.

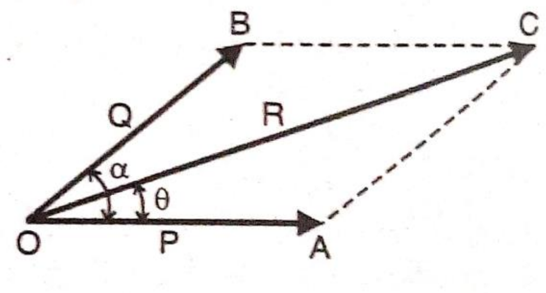



**Fig 2.11**

Forces P and Q form two sides of a parallelogram and according to the law, the diagonal through the point o gives the resultant R as shown.

The magnitude* of resultant is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

The above method of determining the resultant is also known as the cosine law method.

The direction* of the resultant with the force P is given by

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$$

**(b) Graphical method**

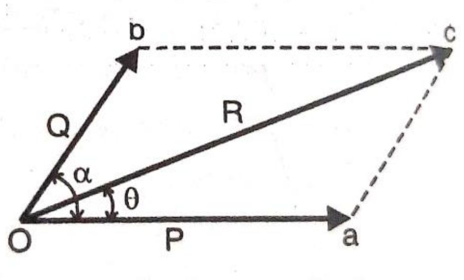



**Fig 2.12**

(i) Choose a convenient scale to represent the forces *P* and *Q*.

(ii) From point *O*, draw a vector *Oa* = *P*

(iii) Now from point *O*, draw another vector *Ob* = *Q* and at an angle of a as shown in Fig. 2.12.

(iv) Complete the parallelogram by drawing lines *ac* ∥ to *Ob* and bc ∥ to *Oa*.

(v) Measure the length *Oc*.

Then resultant R will be equal to length *Oc* × chosen scale.

(vi) Also measure the angle 0, which will give the direction of resultant.

The resultant can also be determine graphically by drawing a triangle *Oac* as explained below and shown in Fig. 2.13.

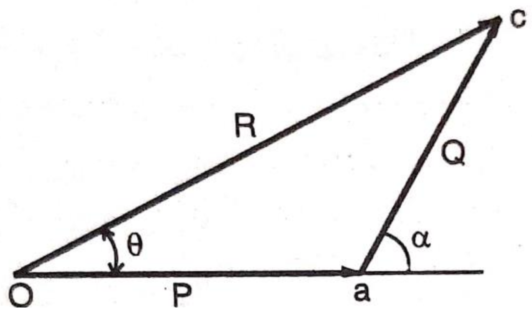



**Fig 2.13**

(i) Draw a line *Oa* parallel to *P* and equal to *P*.

(ii) From *a*, draw a vector ac at an angle $\alpha$ with the horizontal and cut ac equal to *Q*.

(iii) Join *Oc*. Then *Oc* represents the magnitude and direction of resultant R.

Magnitude of resultant *R* = Length *Oc* × chosen scale. The direction of resultant is given by angle 8. Hence measure the angle $\theta$.

### 2.8.2. When More than Two Forces Act at a Point

**(a) Analytical Method**. The resultant of three or more forces acting at a point is found analytically by a method which is known as rectangular components methods (Refer to Art. 1.7). According to this method all the forces acting at a point are resolved into horizontal and vertical components and then algebraic summation *of horizontal and vertical components is done separately. The summation of horizontal component is written as $\sum H$ and that of vertical as $\sum V$. Then resultant R is given by

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

The angle made by the resultant with horizontal is given by

$$\tan\theta = \frac{(\sum V)}{(\sum H)}$$

∴ Let four forces $F_1$, $F_2$, $F_3$ and $F_4$ act at a point $O$ as shown in Fig. 2.14.

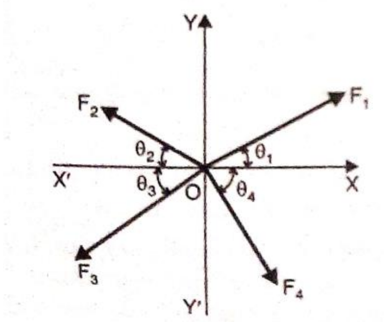



**Fig 2.14**

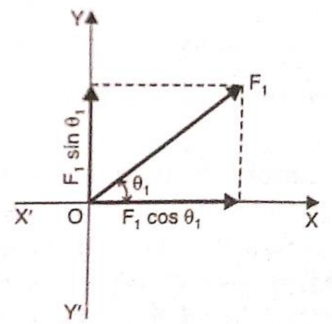



**Fig 2.14(a)**

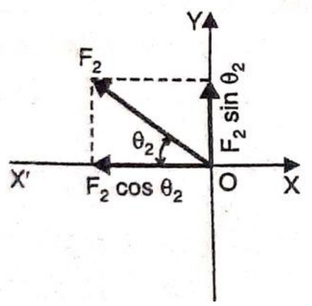



**Fig 2.14(b)**

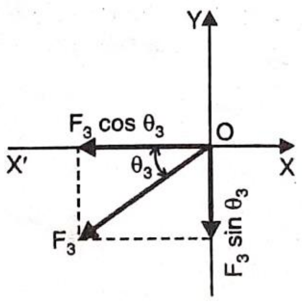



**Fig 2.14(c)**

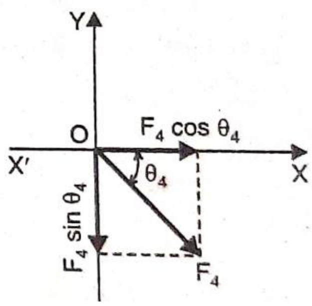



**Fig 2.14(d)**

The inclination of the forces is indicated with respect to horizontal direction, Let

$\theta_1$ = Inclination of force $F_1$ with $OX$

$\theta_2$ = Inclination of force $F_2$ with $OX$

$\theta_3$ = Inclination of force $F_3$ with $OX$

$\theta_4$ = Inclination of force $F_4$ with $OX$

The force $F_1$ is resolved into horizontal and vertical components and these components are shown in Fig. 2.14(a). Similarly, Figs. 2.14(b), (c) and (d) shows the horizontal and vertical components of forces $F_2$, $F_3$, and $F_4$ respectively. The various horizontal components are :

$F_1 \cos \theta_1 \rightarrow (+)$

$F_2 \cos \theta_2 \leftarrow (-)$

$F_3 \cos \theta_3 \leftarrow (-)$

$F_4 \cos \theta_4 \rightarrow (+)$

∴ Summation or algebraic sum of horizontal components :

$\sum V = F_1 \cos \theta_1 - F_2 \cos \theta_2 - F_3 \cos \theta_3 + F_4 \cos \theta_4$

Similarly, various vertical components of all forces are :

$F_1 \sin \theta_1 \uparrow (+)$

$F_2 \cos \theta_2 \uparrow (+)$

$F_3 \cos \theta_3 \downarrow (-)$

$F_4 \cos \theta_4 \downarrow (-)$

∴ 'Summation or algebraic sum of vertical components :

$\sum V = F_1 \sin \theta_1 + F_2 \sin \theta_2 - F_3 \sin \theta_3 - F_4 \sin \theta_4$

Then the resultant will be given by $R = \sqrt{(\sum H)^2 + (\sum V)^2}$ ...(2.1)

And the angle ($\theta$) made by resultant with $x$-axis is given by $\tan \theta = \frac{(\sum V)}{(\sum H)}$

...(2.2)

**(b) Graphical Method.** The resultant of several forces acting at a point is found graphically with the help of the polygon law of forces, which may be stated as

"If a number of coplanar forces are acting at a point such that they can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

Let the four forces $F_1$, $F_2$, $F_3$ and $F_4$ act at a point $O$ as shown in Fig. 2.15. The resultant is obtained graphically by drawing polygon of forces as explained below and shown in Fig. 2.15(a).

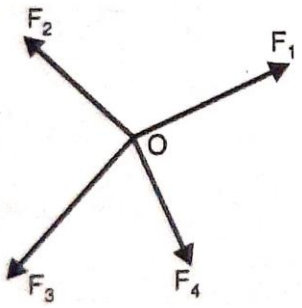



**Fig. 2.15**

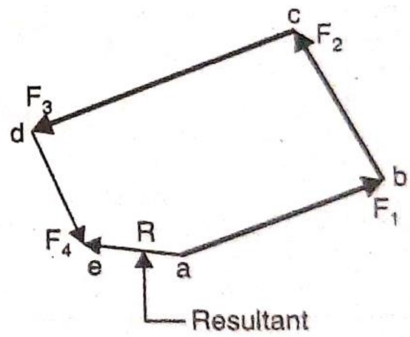



**Fig 2.15(a)**

(i) Choose a suitable scale to represent the given forces.

(ii) Take any point *a*. From *a*, draw vector ab parallel to $OF_1$. Cut *ab* = force $F_1$ to the scale.

(ii) From point *b*, draw *bc* parallel to $OF_2$. Cut *bc* = force $F_2$.

(iv) From point C, draw *cd* parallel to $OF_3$. Cut *cd* = force $F_3$.

(v) From point *d*, draw *de* parallel to $OF_4$. Cut *de* = force $F_4$.

(vi) Join point *a* to *e*. This is the closing side of the polygon. Hence *ae* represents the resultant in magnitude and direction.

Magnitude of resultant R = Length *ae* x scale."

The resultant is acting from *a* to *e*.

**Problem 2.2.** Two forces of magnitude 240 N and 200 N are acting at a point O as shown in Fig. 2.16. If the angle between the forces is 60°, determine the magnitude of the resultant force. Also determine the angle $\beta$ and $\gamma$ as shown in the figure.

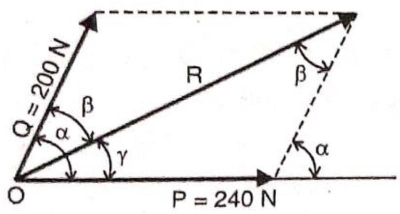



**Fig. 2.16**

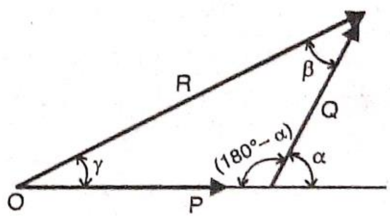



**Fig 2.16(a)**

**Sol.** Given :

Force P = 240 N, Q = 200 N

Angle between the forces, $\alpha$ = 60°

The magnitude of resultant R is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{240^2 + 200^2 + 2 \times 240 \times 200 \times \cos 60°}$$

$$= \sqrt{57600 + 40000 + 48000} = \mathbf{381.57\ N.\ Ans.}$$

Now refer to Fig. 2.16(a). Using sine formula, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)} \qquad \ldots(i)$$

or $$\frac{P}{sin\,\beta} = \frac{R}{sin(180° - \alpha)}$$

$\therefore$ $$\sin\beta = \frac{P\ sin(180° - \alpha)}{R} = \frac{240\ sin(180 - 60)}{381.57}$$

($\therefore$ P = 240 N, $\alpha$ = 60°, R = 381.57 N)

$$= \frac{240 \times sin\,120°}{381.57} = 0.5447$$

$\therefore$ $\beta = \sin^{-1} 0.5447 =$ **33* Ans.**

From equation (i), also we have $\frac{Q}{sin\,\gamma} = \frac{R}{sin(180° - \alpha)}$

$\therefore$ $$\sin\gamma = \frac{Q\ sin(180 - \alpha)}{R}$$

$$= \frac{200\ sin(180 - 60)}{381.57} = \frac{200 \times sin\,120°}{381.57} = 0.4539$$

$\therefore$ $\gamma = \sin^{-1} 0.4539 =$ **26.966* Ans.**

**Problem 2.3.** Two forces P and Q are acting at a point Q as shown in Fig. 2.17. The resultant force is 400 N and angles Band y are 35° and 25° respectively. Find the two forces P and Q.

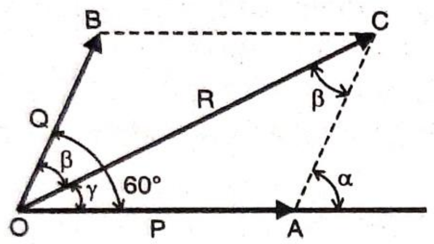



**Fig. 2.17**

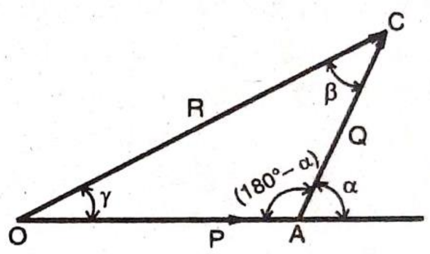



**Fig 2.17(a)**

**Sol.** Given :

Resultant, R = 400 N

Angles, β = 35°, γ = 25°

$\therefore$ Angle between the two forces, $\alpha = \beta + \gamma = 35° + 25° = 60°$

Refer to Fig. 2.17 (a). Using sine formula for ΔOAC, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)} \quad \ldots(i)$$

$$\frac{P}{\sin\beta} = \frac{R}{\sin(180° - \alpha)}$$

$$\therefore \quad P = \frac{R\sin\beta}{\sin(180° - \alpha)} = \frac{240 \times \sin 35°}{\sin(180° - 60°)} \quad (\therefore R = 400, \beta = 35, \alpha = 60°)$$

$$= \frac{400 \times 0.5736}{0.866} = \mathbf{264.93\ N.\ Ans.}$$

Also from equation (i), we have $\frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)}$

$$\therefore \quad Q = \frac{R\sin\gamma}{\sin(180° - \alpha)} = \frac{400 \times \sin 25°}{\sin(180° - 60°)} = \frac{400 \times 0.4226}{0.866}$$

$$= \mathbf{195.19\ N.\ Ans.}$$

**Problem 2.4.** Two forces P and Q are acting at a point O as shown in Fig. 2.18. The force P = 240 N and force Q = 200 N. If the resultant of the forces is equal to 400 N, then find the values of angles β, γ and α.

**Sol.** Given :

Forces, P = 240 N, Q = 200 N

Resultant, R = 400 N

Let β = Angle between R and Q,

γ = Angle between Rand P.

From Fig. 2.18, it is clear that, α = β + γ.

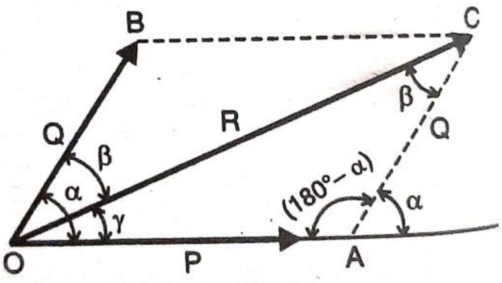



**Fig 2.18**

Let us first calculate the angle a (i.e., Angle between the two forces).

Using the relation,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \text{ or } R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

or $\quad 4002 = 2402 + 2002 + 2 \times 240 \times 200 \times \cos\alpha$

or $\quad 16000 = 57600 + 40000 + 96000 \times \cos\alpha$.

$$\therefore \quad \cos\alpha = \frac{16000-57600-40000}{960000} = 0.65$$

$$\therefore \quad \alpha = \cos^{-1} 0.65 = 49.458° = 49° \ (0{,}458 \times 60') = 49°\ 27.5'$$

Now using sine formula for ΔOAC of Fig. 2.18, we get

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)} \qquad \ldots(i)$$

or
$$\frac{P}{\sin\beta} = \frac{R}{\sin(180° - \alpha)}$$

$$\therefore \quad \sin\beta = \frac{P\ \sin(180° - \alpha)}{R} = \frac{240\ \sin(180° - 49.458)}{400}$$

$$(\because P = 240,\ \alpha = 49.458°)$$

$$= \frac{240\ \sin(130.542°)}{400} = 0.4559$$

$$\therefore \quad \beta = \sin^{-1} 0.4559 = \mathbf{27.12^{*}\ Ans.}$$

Also from equation (i), we have $\frac{Q}{\sin\gamma} = \frac{R}{\sin(180° - \alpha)}$

$$\therefore \quad \sin\gamma = \frac{Q\ \sin(180° - \alpha)}{R} = \frac{200\ \sin(180° - 49.458°)}{400}$$

$$\therefore \quad \gamma = \sin^{-1} 0.3799 = \mathbf{22.33^{*}\ Ans.}$$

**Problem 2.5.** A force of 100 N is acting at a point making an angle of 30° with the horizontal. Determine the components of this force along X and Y directions.

**Sol.** Given :

Force, $\quad F = 100$ N

Angle made by F with horizontal, $\quad \theta = 30°$

Let $\quad F_x$ = Component along $x$-axis

$\quad F_y$ = Component along y-axis

Then $\quad F_x = F \cos\theta = 100 \cos 30°$

$= 100 \times 0.866$

= **86.6 N. Ans.**

and $\quad F_y = F \cos\theta = 100 \cos 30°$

$= 100 \times 0.5 =$ **50 N. Ans.**

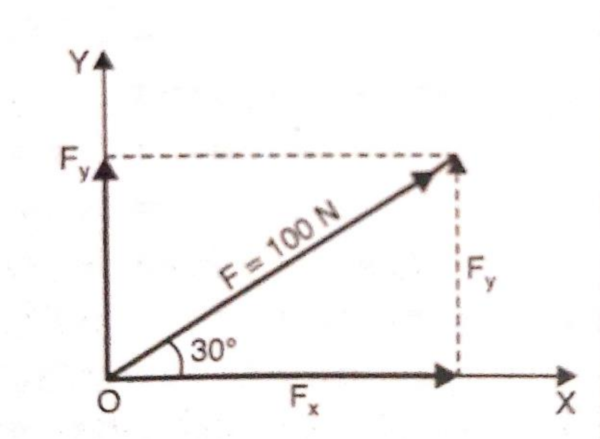



**Fig 2.19**

**Problem 2.6.** A small block of weight 100 N is placed on an inclined plane which makes an angle θ = 30° with the horizontal. What is the component of this weight; (i) parallel to the inclined plane and (ii) perpendicular to the inclined plane?

**Sol.** Given :

Weight of block, $W = 100$ N

Inclination of plane, $\theta = 30°$

The weight of block W = 100 N is acting vertically downwards through the C.G, of the block, Resolve this weight into two components i.e., one perpendicular to the inclined plane and other parallel to the inclined plane is shown in Fig. 2.20. The perpendicular (normal) component makes an angle of 30" with the direction of W.

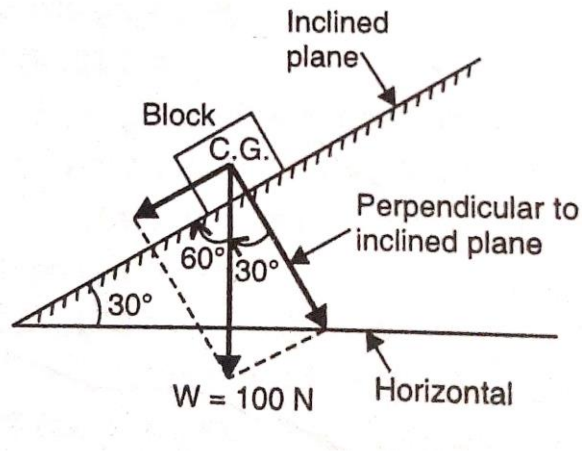



**Fig 2.20**

Hence component of the weight perpendicular to the inclined plane

$= W \cos 30° = 100 \times 0.866 =$ **8.66 N. Ans.**

Component of the weight (W) parallel to the inclined plane

= W sin 30° = 100 × 0.5 = **50 N. Ans.**

**Problem 2.7.** The four coplanar forces are acting at a point as shown in Fig. 2.21. Determine the resultant in magnitude and direction analytically and graphically.

**Sol.** Given :

Forces, $F_1$ = 104 N,

$F_2$ = 156 N,

$F_3$ = 252 N and

$F_4$ = 228 N.

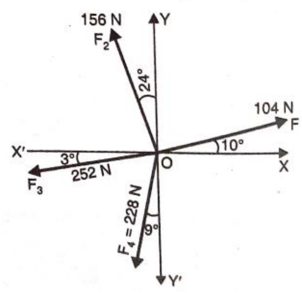



**Fig 2.21**

(a) Analytical Method. Resolve each force along horizontal and vertical axes. The horizontal components along OX will be considered as +ve whereas along OX as -ve. Similarly, vertical components in upward direction will be +ve whereas in downward direction as-ve.

(i) Consider force $F_1$ = 104 N. Horizontal and vertical components are shown in Fig. 2.21 (a).

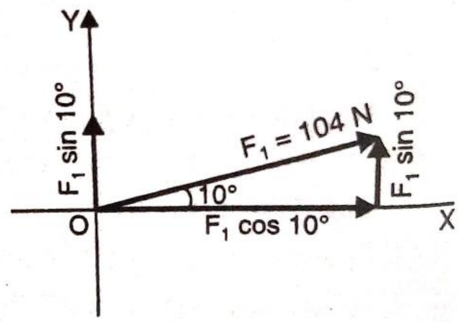



**Fig 2.21(a)**

Horizontal component,

$Fx_1 = F_1 \cos 10° = 104 \times 0.9848$

$= 102.42$ N

Vertical component,

$Fy_1 = F_1 \sin 10° = 104 \times 0.1736$

$= 18.06$ N.

(ii) Consider force $F_2$ = 156 N. Horizontal and vertical components are shown in Fig. 2.21 (b).

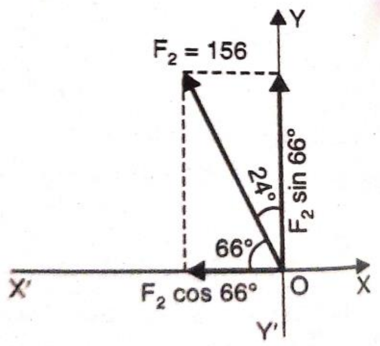



**Fig 2.21(b)**

Angle made by $F_2$ with horizontal axis

OX' = 90 – 24 = 66°

∴ Horizontal components,

$Fx_2 = F_2 \cos 66° = 156 \times 0.4067$

$= 63.44$ N.

It is negative as it is acting along OX'.

Vertical component,

$Fy_2 = F_2 \sin 66° = 156 \times 0.9135$

$= 142.50$ N. (+ve)

(iii) Consider force $F_3$ = 252 N. Horizontal and Vertical components are shown in Fig. 2.21 (c).

Horizontal component,

$Fx_3 = F_3 \cos 3° = 252 \times 0.9986$

$= 251.64$ N. (-ve)

Vertical component,

$Fy_3 = F_3 \sin 3° = 252 \times 0.0523$

$= 13.18$ N. (-ve)

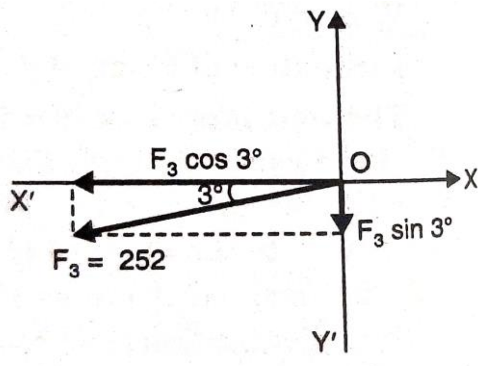



**Fig 2.21(c)**

(iv) Consider force F = 228 N. Horizontal and vertical components are shown in Fig. 2.21 (d).

Angle made by $F_4$ with horizontal axis

OX' = 90 – 9 = 81°

∴ Horizontal components,

$F x_4 = F_4 \cos 81° = 228 \times 0.1564$

$= 35.66$ N. (-ve)

Vertical component,

$F y_4 = F_4 \sin 81° = 228 \times 0.9877$

$= 225.2$ N. (-ve)

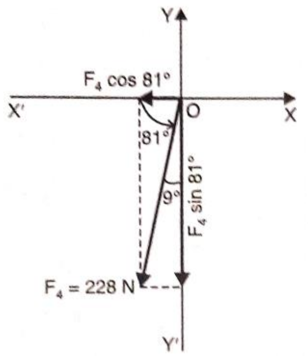



**Fig 2.21(d)**

Now algebraic sum of horizontal components is given by,

$\sum H = F x_1 - F x_2 - F x_3 - F x_4$

$= 102.4 - 63.44 - 251.64 - 35.66$

$= -248.32$ N.

-ve sign means that $\sum H$ is acting along OX' as shown in Fig. 2.21 (e).

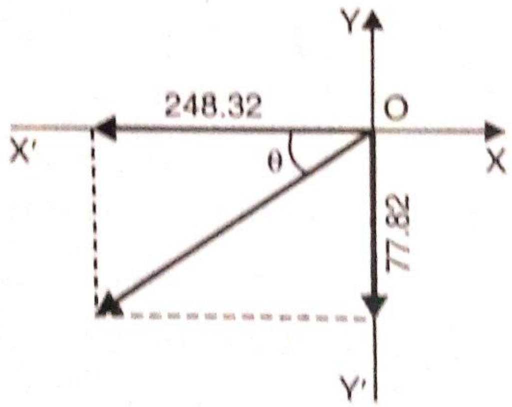



**Fig 2.21(e)**

Similarly, the algebraic sum of vertical components is given by,

$\sum H = 18.06 = 142.50 + 13.18 - 225.2$

$= 77.82$ N.

-ve sign means that EV is acting along OY as shown in Fig. 2.2) (e).

The magnitude of resultant (i.e., R) is obtained by using equation (2.1).

$R = \sqrt{(\sum H)^2 + (\sum V)^2}$

$= \sqrt{(248.32)^2 + (77.82)^2}$ = **260.2 N. Ans.**

The direction of resultant is given by equation (2.2).

$\therefore \qquad \tan\theta = \frac{\sum V}{\sum H} = \frac{77.82}{248.32} = 0.3134$

$\therefore \qquad \theta = \tan^{-1} 0.3134 =$ **17.4°. Ans.**

**(b) Graphical Method.** Fig. 2.22 (a), shows the point at which four forces 104 N, 156 N, 252 N and 228 N are acting. The resultant force is obtained graphically by drawing polygon of forces as explained below and shown in Fig. 2.22 (b):

(i) Choose a suitable scale to represent the given forces. Let the scale is 25 N = 1 cm. Hence the force 104 N will be represented by $\frac{104}{25}$ = 4.16 cm, force 156 N will be represented by $\frac{156}{25}$ = 6.24 cm force 252 N will be represented by $\frac{252}{25}$ = 10.08 cm and the force 228 N will be represented by $\frac{228}{25}$ = 9.12 cm.

(ii) Take any point a. From point *a*, draw vector *ab* parallel to line of action of force 104 N. Cut *ab* = 4.16 cm. Then *ab* represents the force 104 N in magnitude and direction.

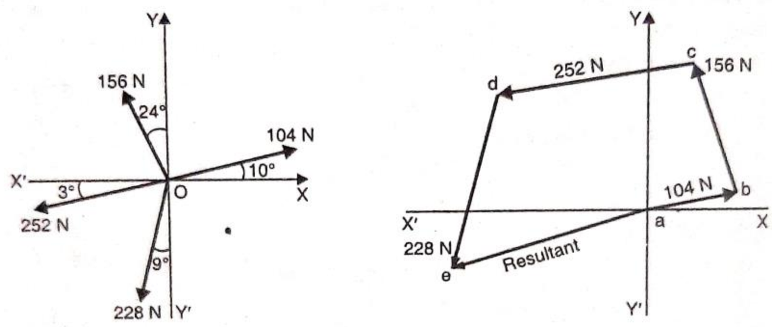



**Fig 2.22(a)** **Fig 2.22(b)**

(iii) From point *b*, draw vector *bc* parallel to force 156 N and cut *bc* = 6.24 cm Then vector cd represents the force 156 N in magnitude and direction.

(iv) From point *c*, draw a vector cd parallel 252 N force and cut *cd* = 10.08 cm. Then vector cd represents the force 252 N in magnitude and direction.

(v) Now from point *d*, draw the vector de parallel to 228 N force and cut *de* = 9.12 cm. Then vector de represents the force 228 N in magnitude and direction.

(vi) Join point *a* to *e*. The line ae is the closing side of the polygon. Hence the side *ae* represents the resultant in magnitude and direction. Measure the length of *ae*.

By measurement, length *ae* = 10.4 cm

$\therefore$ Resultant, R = Length *ae* × Scale = 10.4 × 25

($\because$ 1 cm = 25 N)

Now measure angle made by *ae* with horizontal. This angle is 17.4° with axis OX. **Ans.**

**Problem 2.8.** The resultant of four forces which are acting at a point O as shown in Fig. 2.23, is along Y-axis. The magnitude of forces $F_1$ $F_2$ and $F_3$ are 10 kN, 20 kN and 40 kN respectively. The angles made by 10 kN, 20 kN and 40 kN with X-axis are 30°, 90° and 120° respectively. Find the magnitude and direction of force $F_2$ if resultant is 72 kN.

**Sol.** Given :

$F_1 = 10$ kN, $\theta_1 = 30°$

$F_2 = ?$ $\theta_2 = \theta$

$F_3 = 20$ kN, $\theta_3 = 90°$

$F_4 = 40$ kN, $\theta_4 = 120°$

Resultant, R = 72 kN

Resultant is along Y-axis.

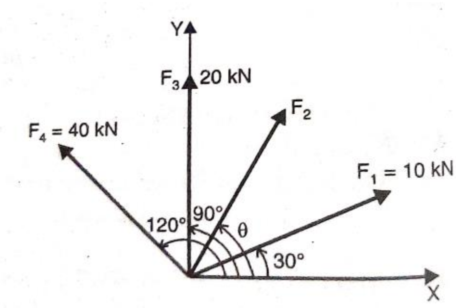



**Fig 2.23**

Hence the algebraic sum of horizontal component should be zero and algebraic sum of vertical components should be equal to the resultant.

$\therefore$ $\sum H = 0$ and $\sum V = R = 72$ KN

But $\sum H = F_1 \cos 30° + F_2 \cos\theta + F_3 \cos 90° + F_4 \cos 120°$

$$= 10 \times 0.866 + F_2 \cos\theta + 20 \times 0 + 40 \times \left(-\frac{1}{2}\right)$$

$$= 8.66 + F_2 \cos\theta + 0 - 20$$

$$= F_2 \cos\theta - 11.34 \quad \ldots\text{(i)}$$

$\therefore$ $\sum H = 0$ or $F_2 \cos\theta - 11.34 = 0$

or $F_2 \cos\theta = 11.34$

Now, $\sum H = F_1 \cos 30° + F_2 \sin\theta + F_3 \sin 90° + F_4 \sin 120°$

$$= 10 \times \frac{1}{2} + F_2 \sin\theta + 20 \times 1 + 40 \times 0.866$$

$= 5 + F_2 \sin\theta + 20 + 34.64$

$= F_2 \sin\theta + 59.64$

But $\qquad \sum H = R$

$\therefore \qquad F_2 \sin\theta + 59.64 = 72$

$\therefore \qquad F_2 \sin\theta = 72 - 59.64 = 12.36 \qquad \ldots$(ii)

Dividing equation (ii) and (i),

$$\frac{F_2 \sin\theta}{F_2 \cos\theta} = \frac{12.36}{11.34} \qquad \text{or} \qquad \tan q = 1.0899$$

$\therefore \qquad \theta = \tan^{-1} 1.0899 =$ **47.46°. Ans.**

Substituting the value of θ in equation (ii), we get $F_2 \sin(47.46°) = 12.36$

or $\qquad F_2 = \dfrac{12.36}{\sin(47.46°)} = \dfrac{12.36}{0.7368} =$ **16.77 kN. Ans.**

## 2.9 ACTION AND REACTION

From the Newton's third law of motion, we know that to every action there is equal and opposite reaction. Hence reaction is always equal and opposite to the action.

Fig. 2.24 (a) shows a ball placed on a horizontal surface (or horizontal plane) such that it is free to move along the plane but cannot move vertically downward. The ball presses the plane downward with a force equal to its own weight and the plane in turn must exert an equal upward force on the ball, according to Newton's third law of motion. Hence the ball will exert a force vertically downwards at the support as shown in Fig. 2.24(b). This force is known as action. The support will exert an equal force vertically upwards on the ball at the point of contact as shown in Fig. 2.24(c).

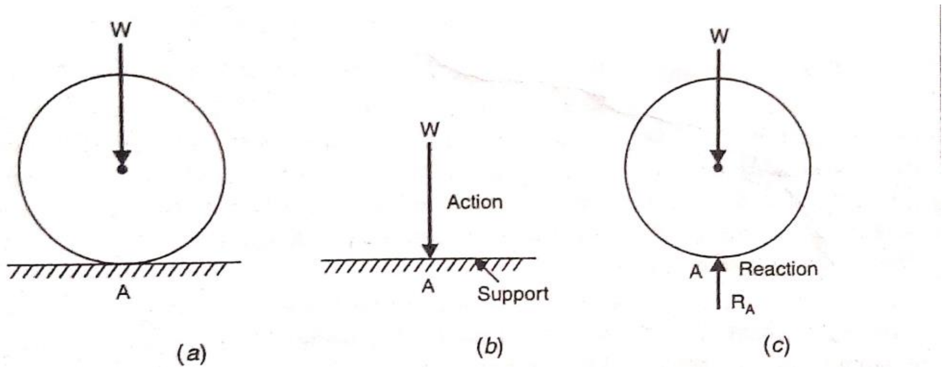



**Fig 2.24**

The force, exerted by the support on the ball, is known as reaction. Hence any force on a support causes an equal and opposite force from the support so that action and reaction are two equal and opposite forces'.

## 2.10 FREE BODY DIAGRAM

The equilibrium of the bodies which are placed on the supports can be considered if we remove the supports and replace them by the reactions which they exert on the body. In Fig. 2.24 (a), if we remove the supporting surface and replace it by the reaction $R_A$ that the surface exerts on the balls as shown in Fig. 2.24 (c), we shall get free-body diagram.

The point of application of the reaction $R_A$ will be the point of contact A and from the law of equilibrium of two forces, we conclude that the reaction $R_A$ must be vertical and equal to the weight W.

Hence Fig. 2.24 (c), in which the ball is completely isolated from its support and in which all forces acting on the ball are shown by vectors, is known a free-body diagram. Hence to draw the free-body diagram of a body we remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these support exert on the body. Also the body should be completely isolated.

**Problem 2.9.** Draw the free body diagram of ball of weight W supported by a string AB and resting against a smooth vertical wall at C as shown in Fig. 2.25 (a).

**Sol.** Given :

Weight of ball = W

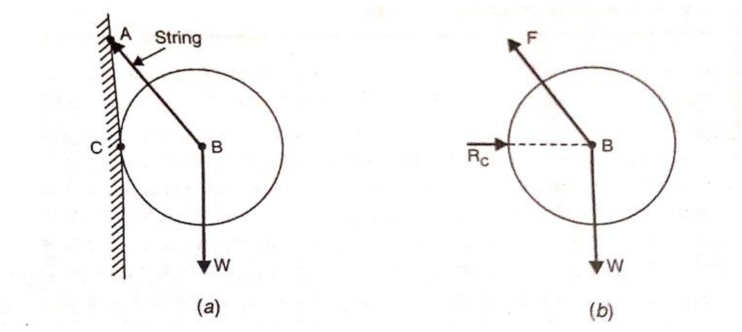



**Fig 2.25**

The ball is supported by a string AB and is resting against a vertical wall at C.

To draw the free-body diagram of the ball, isolate the ball completely (i.e., isolate the ball from the support and string). Then besides the weight W acting at B, we have two reactive forces to apply one replacing the string AB and another replacing the vertical wall AC. Since the string is attached to the ball at B and since a string can pull only along its length, we have the reactive force F applied at B and parallel to BA. The magnitude of F is unknown.

The reaction $R_C$ will be acting at the point of contact of the ball with vertical wall i.e., at point C. As the surface of the wall is perfectly smooth*, the reaction $R_C$ will be normal to the vertical wall (i.e., reaction $R_C$ will be horizontal in this case) und will pass through the point B. The magnitude of $R_C$ is also unknown. The complete free-body diagram is shown in Fig. 2.25(b).

**Problem 2.10**. A circular roller of weight 100 N and radius 10 cm hangs by a tie rod AB = 20 cm and rests against a smooth vertical wall at C as shown in Fig. 2.26 (a). Determine : (i) the force F in the tie rod and (ii) the reaction $R_C$ at point C.

**Sol.** Given :

Weight of roller, $W = 100$ N

Radius of roller, $BC = 10$ cm

Length of tie rod, $AB = 20$ cm

From $\Delta$ABC, we get $\sin\theta = \frac{BC}{AB} = \frac{10}{20}\ 0.5$

$\therefore$ $\theta = \sin^{-1} 0.5 = 30°$

The free-body diagram of the roller is shown in Fig. 2.26 (b) in which

$R_C$ = Reaction at C

F = Force in the tie rod AB

Free-body diagram shows the equilibrium of the roller. Hence the resultant force in $x$-direction and $y$-direction should be zero.

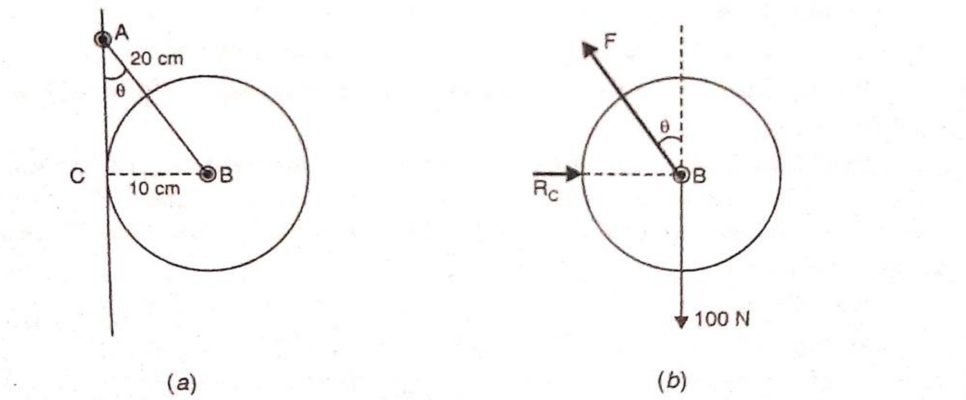



**Fig 2.26**

For $\sum F_x = 0$, we get $R_C - F\sin\theta = 0$

or $R_C = F\sin\theta$

For $\sum F_y = 0$, we get 100 - F cos θ = 0

or 100 = F cos θ

or $F = \frac{100}{\cos\theta} = \frac{100}{\cos 30°}$ ($\because$ θ = 30°)

= **115.47 N. Ans.**

Substituting the value of F in equation (i),

$R_C$= 115.47 × sin 30° = **57.73 N. Ans.**

**Problem 2.11.** Draw the free-body diagram of a ball of weight W, supported by a string AB and resting against a smooth vertical wall at C and also resting against a smooth horizontal floor at D as shown in Fig. 2.27 (a).

**Sol.** Given :

To draw the free-body diagram of the ball, the ball should be isolated completely from the vertical support, horizontal support and string AB. Then the forces acting on the isolated ball as shown in Fig. 2.27 (b), will be :

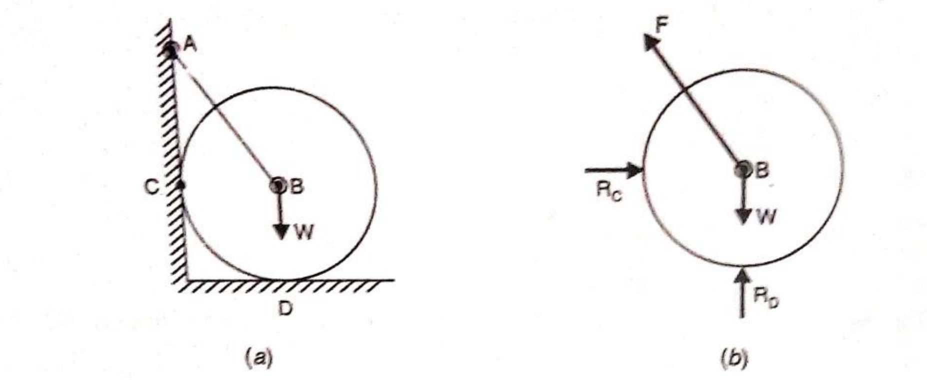



**Fig 2.27**

(i) Reaction $R_C$ at point C, normal to AC.

(ii) Force Fin the direction of string.

(iii) Weight W of the ball.

(iv) Reaction $R_D$ at point D, normal to horizontal surface.

The reactions $R_C$ and $R_D$ will pass through the centre of the ball *i.e.,* through point B.

**Problem 2.12.** A ball of weight 120 N rests in a right-angled groove, as shown in Fig. 2.28 (a). The sides of the groove are inclined to an angle of 30° and 60° to the horizontal. If all the surfaces are smooth, then determine the reactions $R_A$ and $R_C$ at the points of contact.

**Sol.** Given:

Weight of ball, W = 120 N

Angle of groove = 90°

Angle made by side FD with horizontal = 30°

Angle made by side ED with horizontal = 60°

∴ Angle FDH = 30° and angle EDG = 60°

Consider the equilibrium of the ball. For this draw the free body diagram of the ball as shown in Fig. 2.28(b).

The forces acting on the isolated ball will be:

(i) Weight of the ball = 120 N and acting vertically downwards.

(ii) Reaction $R_C$ acting at C and normal to FD.

(iii) Reaction $R_A$ acting at A and normal to DE.

The reactions $R_A$ and $R_C$ will pass through B, i.e., centre of the ball. The angles made by $R_A$ and $R_C$ at point B will be obtained as shown in Fig. 2.28(c).

In ΔHDC, ∠CDH = 30° and ∠DCH = 90°. Hence ∠DHC will be 60°. Now in ∠HBL, ∠BLH = 90° and angle LHB= 60°. Hence ∠HBL will be 30°.

Similarly, ∠GBL may be calculated. This will be equal to 60°.

For the equilibrium of the ball,

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

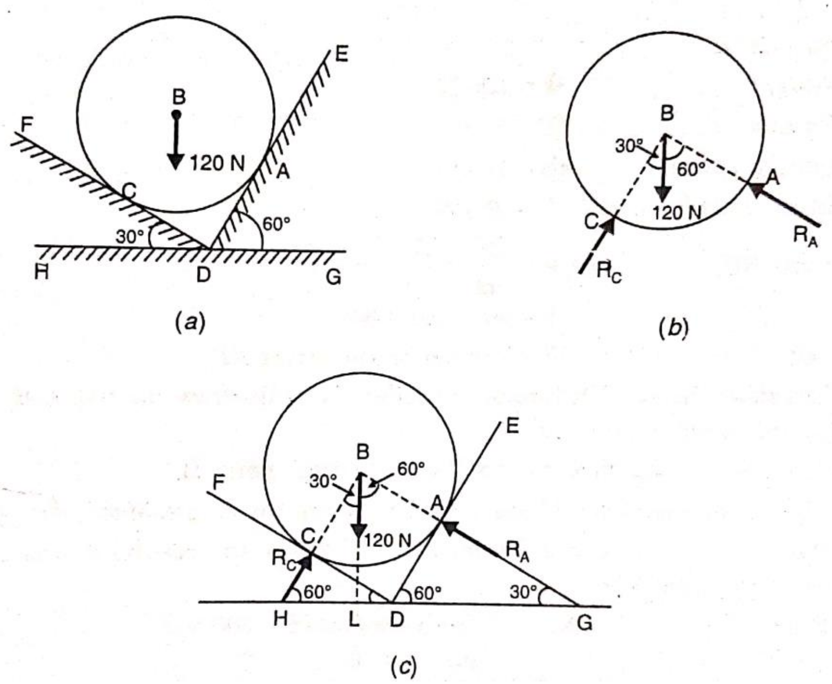



**Fig 2.28**

For $\sum F_x = 0$, we have $R_C \sin 30° - R_A \sin 60° = 0$

or $R_C \sin 30° = R_A \sin 60°$

or $$R_C = R_A \times \frac{0.866}{\sin 30°} = 1.732\ R_A \quad \ldots(i)$$

For $\sum F_y = 0$, we have $120 - R_A \sin 60° - R_C \sin 30° = 0$

or $$120 = R_A \cos 60° + R_c \cos 30°$$

$$= R_A \times 0.5 + (1.732\ R_A) \times 0.866 \qquad (\because R_C = 1.732\ R_A)$$

$$= 0.5\ R_A + 1.5\ R_A = 2\ R_A.$$

$\therefore$ $R_A = \frac{120}{2} =$ **60 N. Ans.**

Substituting this value in equation (i), we get

$R_C = 1.732 \times 60 =$ **103.92 N. Ans.**

**Problem 2.13.** A circular roller of radius 5 cm and of weight 100 N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 10 cm as shown in Fig. 2.29. A horizontal force of 200 N is acting at B. Find the tension (or Forces) in the bar AB and the vertical reaction at C.

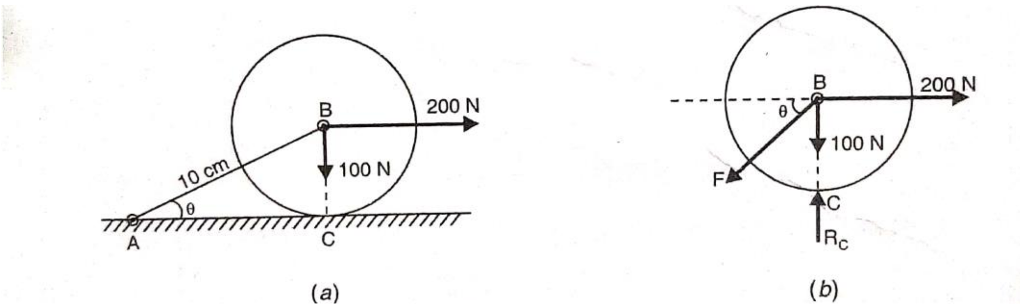



**Fig 2.29**

**Sol.** Given:

Weight, $W = 100$ N

Radius i.e., $BC = 5$ cm

Length of bar, $AB = 10$ cm

Horizontal force at $B = 200$ N

In $\Delta ABC$, $\sin \theta = \frac{BC}{AB} = \frac{5}{10}\ 0.5$

$\therefore$ $\theta = \sin^{-1} 0.5 = 30°$

Let, $F$ = Tension in the string AB.

Consider the equilibrium of the roller. For this draw the free body diagram of the roller as shown in Fig. 2.29 (6).

The reaction $R_C$ at point C will pass through point B.

The tension (or force F) will be acting along the length of the string.

As the roller is in equilibrium in Fig. 2.29 (b), the resultant force in $x$-direction and $y$-direction should be zero.

For $\sum F_x = 0$, we have $F \cos \theta - 200 = 0$

$\therefore$ $F = \frac{200}{\cos \theta} = \frac{200}{\cos 30°}$ $(\therefore \theta = 30°)$

$= \mathbf{230.94\ N.\ Ans.}$

For $\sum F_x = 0$, we have $R_C - W - F \sin \theta = 0$

or $R_C = W + F \sin \theta = 100 + 230.94 \times \sin 30$

$= \mathbf{215.47\ N.\ Ans.}$

## STUDENT ACTIVITY

**1.** What is the difference between collinear and concurrent forces?

**2.** Explain the procedure of resolving a given force into two components at right angles to each other.

## SUMMARY

1. Coplanar forces means the forces are acting in one plane.
2. Concurrent forces means the forces are intersecting at a common point.
3. Collinear forces means the forces are having some line of action.
4. The resultant of coplanar forces are determined by analytical and graphical methods.
5. The resultant (R) of three collinear forces $F_1$, $F_2$ and $F_3$ acting in the same direction, is given by R = $F_1 + F_2 + F_3$. If the force F, is acting in opposite direction then their resultant will be, R = $F_1 + F_2 + F_3$.
6. The resultant of the two forces P and Q having an angle $\alpha$ between them and acting at a point, is given by cosine law method as R = $\sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$. And the direction of the resultant with the force P is given by,
7. The resultant of three or more forces acting at a point is given by, R = $\sqrt{(\sum H)^2 + (\sum V)^2}$ , where $\sum H$ = Algebraic sum of horizontal components of all forces, $\sum V$ = Algebraic sum of vertical components of all forces. The angle made by the resultant with horizontal is given by, tan θ = $\frac{(\sum H)}{(\sum V)}$
8. The resultant of several forces acting at a point is found graphically by using polygon law of forces.
9. Polygon law of forces states that if a number of coplanar forces are acting at a point such that they can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

## TEST YOURSELF

### (A) Theoretical Problems

1. Define and explain the following terms :
   (i) Coplanar and non-coplanar forces
   (ii) Collinear and concurrent forces
   (iii) Parallel and non-parallel forces.
2. What is the difference between collinear and concurrent forces ?
3. State and explain the following laws of forces :
   (i) Law of parallelogram of forces
   (ii) Law of triangle of forces
   (iii) Law of polygon of forces.
4. Derive an expression for the resultant in magnitude and direction of two coplanar concurrent forces using cosine law method.

5. Explain in detail the method of finding resultant in magnitude and direction of three or more forces acting at a point by analytical and graphical method.
6. Explain the procedure of resolving a given force into two components at right angles to each other.
7. Three collinear forces $F_1$, $F_2$ and $F_3$ are acting on a body. What will be the resultant of these forces, if
   (a) all are acting in the same direction
   (b) force $F_3$ is acting in opposite direction.
8. State the law of parallelogram of forces and show that the resultant $R = \sqrt{P^2 + Q^2}$ when the two forces P and Q are acting at right angles to each other. Find the value of R if the angle between the forces is zero.

**(B) Numerical Problems**

**1.** Three collinear horizontal forces of magnitude 300 N, 100 N and 250 N are acting on rigid body. Determine the resultant of the forces analytically and graphically when : (i) all the forces are acting in the same direction ; (ii) the force 100 N acts in the opposite direction.

[**Ans.** (i) 650 N, (ii) 450 N]

**2.** Two forces of magnitude 15 N and 12 N are acting at a point. The angle between the forces is 60°. Find the resultant is magnitude.

[Ans. 20.43 N]

**3.** A force of 1000 N is acting at a point, making an angle of 60° with the horizontal. Determine the components of this force along horizontal and vertical directions.

[**Ans.** 500 N, 866 N]

**4.** A small block of weight 100 N is placed on an inclined plane which makes an angle of 60° with the horizontal. Find the components of this weight (i) perpendicular to the inclined plane and (ii) parallel to the inclined plane.

[**Ans.** 50 N, 86.6 N]

**5.** Two forces P and Q are acting at a point O as shown in Fig. 2.30. The force P = 264.9 N and force Q = 195.2 N. If the resultant of the forces is equal to 400 N then find the values of angles $\beta$, $\gamma$, $\alpha$.

[**Ans.** $\beta = 35°$, $\gamma = 25°$, $\alpha = 60°$]

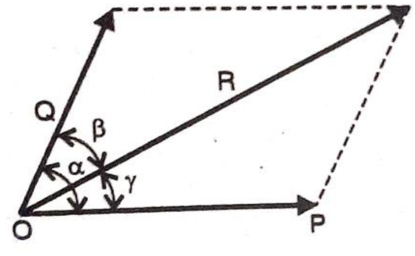



**Fig 2.30**

**6.** A small block of unknown weight is placed on an inclined plane which makes an angle of 30° with horizontal plane. The component of this weight parallel to the inclined plane is 100 N. Find the weight of the block.

[**Ans**. 200 N]

**7.** In question 6, find the component of the weight perpendicular to the inclined plane.

[**Ans.** 173.2 N]

**8.** The four coplanar forces are acting at a point as shown in Fig. 2.31. Determine the resultant in magnitude and direction analytically an graphically.

[**Ans.** 1000 N, θ = 60° with OX]

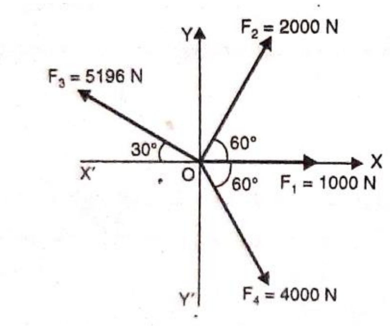



**Fig 2.31**

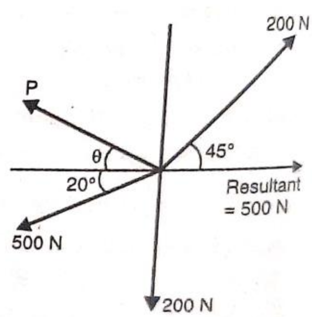



**Fig 2.32**

**9.** The four coplanar forces are acting at a point as shown as in Fig. 2.32. one of the forces is unknown and its magnitude is shown by P. The resultant is having a magnitude 500 N and is acting along *x*-axis. Determine the unknown force P and its inclination with *x*-axis.

[**Ans.** P = 286.5 N and θ = 53° 15']

# 3 MOMENTS

**LEARNING OBJECTIVES**

- Introduction
- Concept of Moment
- Varignon's Theorem
- Parallel Forces
- Resultant of Two Parallel Forces
- Effect of a Force Moving Parallel to its Line of Action
- General Case of Parallel Forces in a Plane
- Equivalent System
- General Condition of Equilibrium of Bodies under Coplanar Forces

## 3.1 INTRODUCTION

The forces, which are having their line of actions parallel to each other, are known parallel forces. The two parallel forces will not intersect at a point. The resultant of two coplanar concurrent forces (i.e., forces intersecting at the same point) can be directly determined by the method of parallelogram of forces. This method along with other methods for finding resultant of collinear and concurrent coplanar forces, were discussed in earlier chapters.

The parallel forces are having their lines of action parallel to each other. Hence, for finding the resultant of two parallel forces, the parallelogram cannot be drawn. The resultant of such forces can be determined by applying the principle of moments. Hence in this chapter first the concepts of moment and principle of moments will be dealt with. Thereafter the methods of finding resultant of parallel and even non-parallel forces will be explained.

## 3.2 CONCEPT OF MOMENT

The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Let F = A force acting on a body as shown in Fig. 3.1.

$r$ = Perpendicular distance from the point on the line of action of force F.

Then moment (M) of the force F about O is given by,

M =F × T

The tendency of this moment is to rotate the body in the clockwise direction about O. Hence this moment is called clock- wise moment. If the tendency of a moment Point about which is to rotate the body in anti-clockwise direction, then that moment is known as anti-clockwise moment. If clockwise moment is taken -ve then anti-clockwise moment will be +ve.

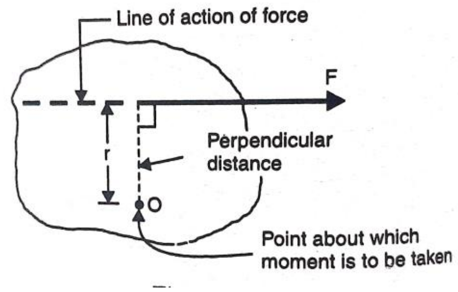



**Fig. 3.1**

In S.I. system, moment is expressed in Nm (Newton metre).

Fig. 3.2 shows a body on which three forces $F_1$ $F_2$ and $F_3$ are acting. Suppose it is required to find the resultant moments of these forces about point O.

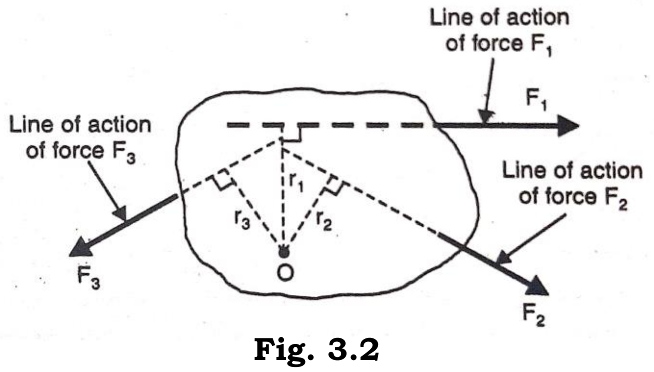



**Fig. 3.2**

Let $r_1$ = Perpendicular distance from O on the line of action of force $F_1$.

$r_2$ and $r_3$ = Perpendicular distances from 0 on the lines of action of force $F_2$.

Moment of $F_1$ about 0 = $F_1 \times r_1$ (clockwise) (-)

Moment of $F_2$ about 0 = $F_2 \times r_2$ (clockwise) (-)

Moment of $F_3$ about 0 = $F_3 \times r_3$ (anti-clockwise) (+)

The resultant moment will be algebraic sum of all the moments.

∴ The resultant moment of $F_1$, $F_2$ and $F_3$ about O

$$= - F_1 \times r_1 - F_2 \times r_2 + F_3 \times r_3$$

**Problem 3.1.** Four forces of magnitude 10 N, 20 N, 30 N and 40 N are acting respectively along the four sides of a square ABCD as shown in Fig. 3.3. Determine the resultant moment about the point A. Each side of the square is given 2 m.

**Sol.** Given :

Length AB = BC = CD = DA = 2 m

Force at B= 10 N,

Force at C = 20 N,

Force at D = 30 N,

Force at A = 40 N,

The resultant moment about point A is to be determined.

The forces at A and B passes through point A. Hence perpendicular distance from A on the lines of action of these forces will be zero.

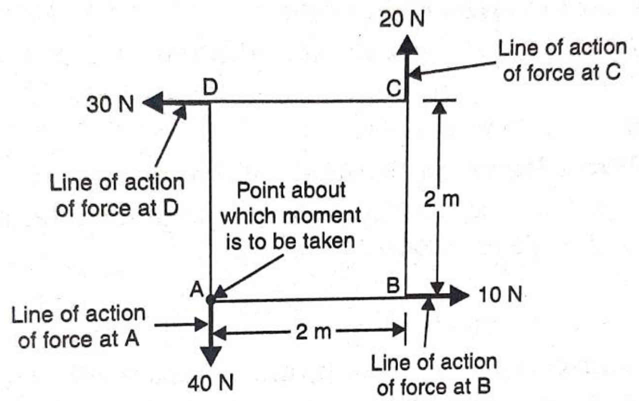



**Fig. 3.3**

Hence their moments about A will be zero. The moment of the force at C about point A.

= Force at C × ⊥ distance from A on the line of action of force at C.

= (20 N) × (Length AB).

= 20 × 2 Nm = 40 Nm (anti-clockwise).

The moment of force at D about point A.

= Force at D × ⊥ distance from A on the line of action of force at D.

= (30 N) × (Length AD).

= 30 × 2 Nm = 60 Nm (anti-clockwise).

∴ Resultant moment of all forces about A.

= 40 + 60 = **100 Nm** (anti-clockwise). **Ans.**

## 3.3 VARIGNON'S THEOREM

**3.3.1. Principle of Moments.** Principle of moments states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

Varignon's theorem states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

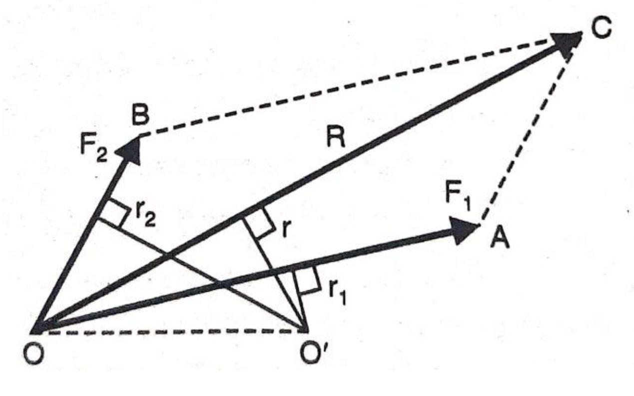



**Fig. 3.4**

Fig. 3.4 Shows two forces $F_1$ and $F_2$ acting at point O. These forces are represented in magnitude and direction by OA and OB. Their resultant R is represented in magnitude and direction by OC which is the diagonal of parallelogram OACB. Let O' is the point in the plane about which moments of $F_1$, $F_2$ and R are to be determined. From point O', draw perpendiculars on OA, OC and OB,

Let $r_1$ Perpendicular distance between $F_1$ and O'.

$r$ = Perpendicular distance between R and O'.

$r_2$ = Perpendicular distance between $F_2$ and O'.

Then according to Varignon's principle;

Moment of R about ό must be equal to algebraic sum of momenta of $F_1$ and $F_2$ about O'.

or $$R \times r = - F_1 \times r_1 + F_2 \times r_2$$

**3.3.2. Problems Based on Principle of Moments**

**Problem 3.2.** A force of 100 N is acting at a point A as shown in Fig. 3,5. Determine the moments of this force about O.

**Sol.** Given :

Force at A = 100 N

Draw a perpendicular from 0 on the line of action of force 100 N. Hence OB is the perpendicular on the line of action of 100 N as shown in Fig. 3.6.

**1st Method**

Triangle OBC is a right-angled triangle. And anle

$$OBC = 60°$$

$$\therefore \quad \sin 60° = \frac{OB}{OC}$$

$$\therefore \quad OB = OC \sin 60°$$

$$= 3 \times 0.866$$

$$= 2.598 \text{ m}$$

Moment of the forces 100 N about O

$$= 100 \times OB = 100 \times 2.598$$

= **259.8 Nm** (clockwise) **Ans.**

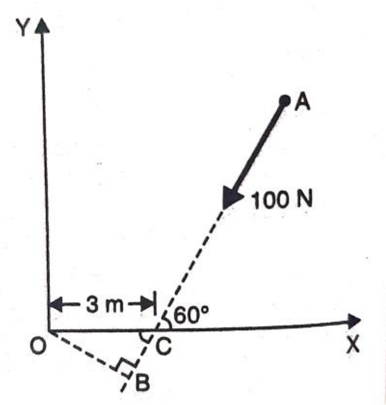



**Fig. 3.5**

**2 Method**

The moment of force 100 N about O, can also be determined by using Varignon's principle. The force 100 N is replaced by its two rectangular components at any convenient point. Here the convenient point is chosen as C. The horizontal and vertical components of force 100 N acting at Care shown in Fig. 3.6.

The horizontal component

$$= 100 \times \cos 60° = 50 \text{ N}$$

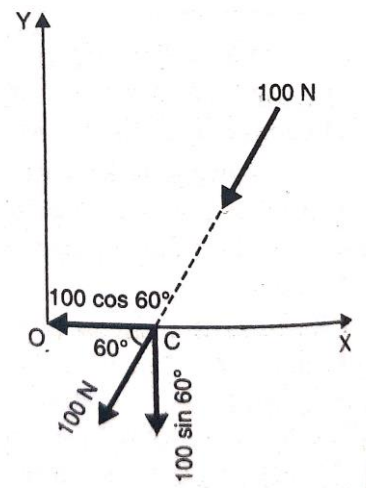



**Fig. 3.6**

But this force is passing through O and hence has no moment about O.

The vertical component

$= 100 \times \cos 60° = = 100 \times 0.866 = 86.6$ N

This force is acting vertical downwards at C. Moment of this force about O.

$= 86{,}6 \times OC = 86.6 \times 3$ $(\therefore OC = 3 \text{ m})$

= **260.8 N** (clockwise). **Ans.**

## 3.4 parallel forces

The following are the important types of parallel forces :

1. Like parallel forces,
2. Unlike parallel forces.

3.4.1. Like Parallel Forces. The parallel forces Which are acting in the same direction, are lowing like parallel forces In fig.3.7, two parallel forces $F_1$ and $F_2$ are shown. They are acting in the same direction. Hence they are called a like parallel forces. There forces may be equal or equal in magnitude.

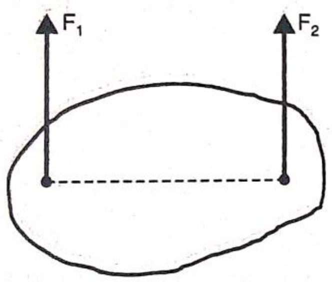



**Fig. 3.7**

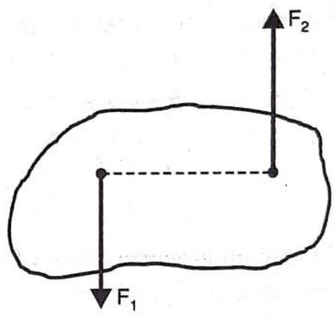



**Fig. 3.8**

**3.4.2. Unlike Parallel Forces.** The parallel forces which are acting in the opposite direction, are known as unlike parallel forces. In Fig. 3.8, two parallel forces $F_1$ and $F_2$ are acting in opposite direction. Hence they are called as unlike parallel forces. These forces may be equal or unequal in magnitude.

The unlike parallel forces may be divided into: (i) unlike equal parallel forces, and (ii) unlike unequal parallel forces.

Unlike equal parallel forces are those which are acting in opposite direction and are equal in magnitude.

## 3.5 RESULTANT OF TWO PARALLEL FORCES

The resultant of following two parallel forces will be considered :

1. Two parallel forces are like.
2. Two parallel forces are unlike and are unequal in magnitude.
3. Two parallel forces are unlike but equal in magnitude.

**3.5.1. Resultant of Two like Parallel Forces.** Fig. 3.9 shows a body on which two like parallel forces, $F_1$ and $F_2$ are acting. It is required to determine the resultant (R) and also the point at which the resultant R is acting. For the two parallel forces which are acting in the name direction, obviously the resultant R le given by,

$$R = F_1 + F_2$$

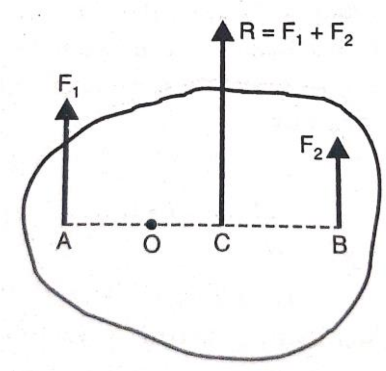



**Fig. 3.9**

In order to find the point At which the resultant is acting, Varignon's principle (or method of moments) is used, According to this, the algebraic sum of moments of $F_1$ and $F_2$ about any point should be equal to the moment of the resultant (R) about that point. Now arbitrarily choose any point o along line AB and take moments of all forces about this point,

Moment of $F_1$ about O = $F_1 \times AO$ (clockwise) (-)

Moment of $F_2$ about O = $F_2 \times BO$ (anti-clockwise) (+ve)

Algebraic sum of moments of $F_1$ and $F_2$ about O

$$= F_1 \times AO + F_2 \times BO$$

Moment of resultant about 0= $R \times OC$ (anti-clockwise)(+)

But according to principle of moments the algebraic sum of moments of $F_1$ and $F_2$ about should be equal to the moment of resultant about the same point O.

$$\therefore \quad - F_1 \times AO + F_2 \times BO = +R \times CO = (F_1 + F_2) \times CO$$

$$(\because R = F_1 + F_2)$$

or $$F_1(AO + CO) = F_2(BO - CO)$$

or $$F_1 \times AO = F_2 \times BC$$

$$(\because AO + CO = AC \text{ and } BO - CO = BC)$$

or $$\frac{F_1}{F_1} = \frac{BC}{AC}$$

The above relation shows that de resultant R acts at the point C, parallel to the lines of action of the given forces $F_1$ and $F_2$ in such a way that the resultant divides the distance AB in the ratio inversely proportional to the magnitudes of $F_1$ and F Also the point C lies in line AB i.e., point C is not outside AB.

The location of the point C, at which the resultant R is acting, can also be deter- mined by taking moments about points A of Fig. 3.9. As the force $F_1$ is passing through A, the moment of $F_1$ about A will be zero.

The moment of $F_2$ about

$$A = F_2 \times AB \text{ (anti-clockwise) } (+)$$

Algebraic sum of moments of $F_1$ and $F_2$ about O

$$= 0 + F_2 \times AB = F_2 \times AB \text{ (anti-clockwise) } (+) \qquad \ldots(i)$$

The moment of resultant R about A

$$= R \times AC \text{ (anti-clockwise)}(+) \qquad \ldots(ii)$$

But according to the principle of moments, the algebraic sum of moments of $F_1$ and $F_2$ about A should be equal to the moment of resultant about the same point A. Hence equating equations (i) and (ii).

$$F_2 \times AB = R \times AC$$

But $R = (F_1 + F_2)$ hence the distance AC should be less than AB. Or in other words, the point C will lie inside AB.

**3.5.2. Resultant of Two Unlike Parallel Forces (Unequal in Magnitude).** Fig 3.10 shows a body on which two unlike parallel forces $F_1$ and $F_2$ are acting which are unequal in magnitude. Let us assume that force $F_1$ is more than $F_2$. It is required to determined the resultant R and also the point at which the resultant R is acting. For the two parallel forces, which are acting in opposite direction, obviously the resultant is given by,

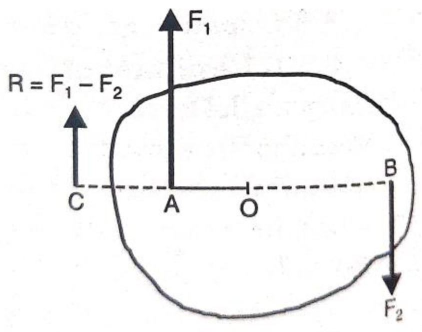



**Fig. 3.10**

$$R = F_1 - F_2$$

Let the resultant R is acting at C as shown in Fig. 3.10.

In order to find the point C, at which the resultant is acting, principle of moments is used.

Choose arbitrarily any point O in line AB. Take the moments of all forces (i.e., $F_1$, $F_2$ and R) about this point.

Moment of $F_1$ about O = $F_1 \times AO$ (clockwise)

Moment of $F_2$ about O = $F_2 \times BO$ (clockwise)

Algebraic sum of moments of $F_1$ and $F_2$ about O

$$= F_1 \times AO + F_2 \times BO \qquad \ldots(i)$$

Moment of resultant force R about O

$$= R \times CO \text{ (clockwise)}$$

$$= (F_1 - F_2) \times CO \qquad (\therefore R = F_1 - F_2)$$

$$= F_1 \times CO - F_2 \times CO \qquad \ldots(ii)$$

But according to the principle of moments, the algebraic sum of moments of all forces about any point should be equal to the moment of resultant about that point. Hence equating equations (i) and (ii), we get

$$F_1 \times AO + F_2 \times BO = F_1 \times CO - F_2 \times CO$$

or $$F_2(BO + CO) = F_1(CO - AO)$$

$$F_2 \times BC = F_1 \times AC \qquad (\therefore BO + CO = BC \text{ and } CO - AO = AC)$$

or $$\frac{BC}{AC} = \frac{F_1}{F_1} \qquad \text{or} \qquad \frac{F_1}{F_1} = \frac{BC}{AC}$$

But $F_1 > F_2$, hence BC will be more than AC. Hence point O lies outside of AB and on the same side as the larger force $F_1$. Thus in case of two unlike parallel forces the resultant lies outside the line joining the points of action of the two forces and on the same side as the larger force.

The location of the point C, at which the resultant R is acting, can also be determined by taking moments about point A, of Fig. 3.10. As the force $F_1$ is passing through A, the moment of $F_1$ about A will be zero.

The moment of $F_2$ about A = $F_2 \times AB$ (clockwise) (-)

Algebraic sum of moments of $F_1$ and $F_2$ about A

$$= O + F_2 \times AB = F_2 \times AB \text{ (clockwise) (-)} \qquad \ldots(i)$$

The moment of resultant R about A should be equal to the algebraic sum of moments of $F_1$ and $F_2$ (i.e., = $F_2 \times AB$) according to the principle of moments. Also the moment of resultant R about A should be clockwise. As R is acting upwards [$\therefore F_1 > F_2$ and $R = (F_1 - F_2)$ so R is acting in the direction of F ], the moment of resultant R about A would be clockwise only if the points C is towards the left of point A. Hence the point C will be outside the line AB and on the side of $F_1$ (i.e., larger force).

Now the moment of resultant R about A

$$= R \times AC \text{ (clockwise) (-)} \qquad \ldots(ii)$$

Equating equations (i) and (ii),

$$F_2 \times AB = R \times AC$$

$$= (F_1 - F_2) \times AC \qquad (\therefore R = F_1 - F_2)$$

As $F_1$, $F_2$ and AB are known, hence AC can be calculated. Or in other words, the location of point C is known.

**3.5.3. Resultant of Two Unlike Parallel Forces which are Equal in Magnitude.** When two equal and opposite parallel forces act on a body, at some distance apart, the two forces from a couple which has a tendency to rotate the body. The perpendicular distance between the parallel forces in known as arm of the couple.

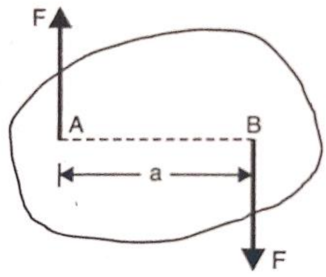



**Fig. 3.11**

Fig. 3.11 shows a body on which two parallel forces, which are acting in opposite direction but equal in magnitude are acing. These two forces will form a couple which will have a tendency to rotate the body in clockwise direction. The moment of the couple is the product of either one of the forces and perpendicular distance between the forces.

Let F = Force at A or at B

$a$ = Perpendicular distance (or arm of the couple)

The moment (M) of the couple is given by, M = F × $a$.

The units of moment will be Nm.

**3.5.4. Problems based on Parallel Forces**

**Problem 3.3.** Three like parallel force 100 N, 200 N and 300 N are acting at points A, B and C respectively on a straight line ABC as shown in Fig. 3.12. The distances are AB = 30 cm and BC = 40 cm. Find the resultant and also the distance of the resultant from point A on line ABC.

**Sol.** Given :

Force at A = 100 N

Force at B = 200 N

Force at C = 100 N

Distance AB = 30 cm, BC = 40 cm. As all the forces are parallel and acting in the same direction, their resultant R is given by

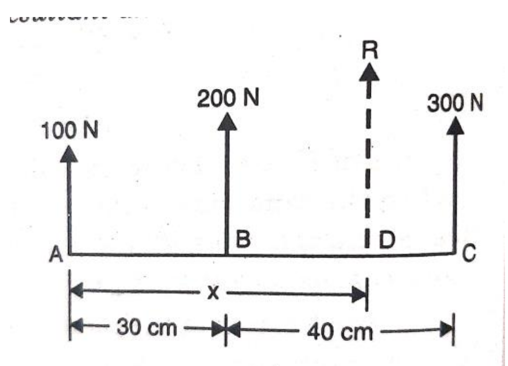



**Fig. 3.12**

$$R = 100 + 200 + 300 = 600 \text{ N}$$

Let the resultant is acting at a distance of $x$ cm from the point A as shown in Fig. 3.12.

Now take the moments of all forces about point A. The force 100 N is passing A, hence its moment about A will be zero.

∴ Moment of 100 N force about A = 0

Moment of 200 N force about A = 200 × 30 = 6000 N cm (anti-clockwise)

Moment of 300 N force about A = 300 × AC

= 300 × 70 = 21000 N cm (anti-clockwise)

Algebraic sum of moments of all forces about A

= 0 + 6000 + 21000 = 27000 N cm (anti-clockwise)

Moment of resultant R about A = R × $x$

= 600 × $x$ N cm (∴ R = 600)

But algebraic sum of moments of all forces about A

= Moment of resultant about A

or $27000 = 600 \times x$ or $x = \frac{27000}{600}$ = **45cm.**

**Problem 3.4.** The three like parallel forces of magnitude 50 N, F and 100 N are shown in Fig. 3.13. If the resultant R = 250 N and is acting at a distance of 4 m from A, then find

(i) Magnitude of force F.
(ii) Distance of F from A.

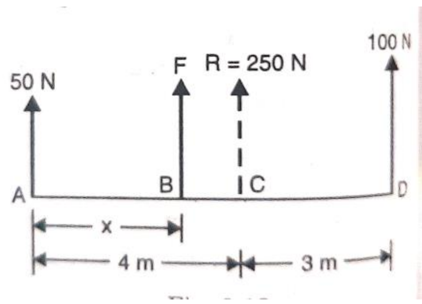


**Fig. 3.13**

**Sol.** Given :

Forces at A = 50 N, at B = F and D = 100 N

R = 250 N,

Distance AC = 4 m, CD = 3 m.

(i) Magnitude of force F

The resultant R of three like forces is given by,

$$R = 50 + F + 100$$

or 250 = 50 + F + 100 (∴ R = 250)

$\therefore$ F = 250 – 50 – 100 = **100 N. Ans.**

(ii) Distance of F from A

Take the moments of all forces about point A.

Moment of force 50 N about A = 0 ($\because$ Force 50 N is passing through)

Moment of force F about A = F × $x$ (anti-clockwise)

Moment of force 100 N about A = 100 AD = 100 × 7 = 700 Nm (anti-clockwise)

$\therefore$ Algebraic sum of moments of all forces about A

= 0 + F × $x$ + 700 Nm

= F × $x$ + 700 Nm (anti-clockwise)

Moment of resultant R about A = R × 4 = 250 × 4 = 1000 Nm (anti-clockwise)

But algebraic sum of moments of all forces about A must be equal to the moment of resultant R about A.

$\therefore$ F × $x$ + 700 = 1000 or F × $x$ = 1000 – 700 = 300

or $x = \frac{300}{F} = \frac{300}{100}$ ($\because$ F = 100 N)

= **3 m. Ans.**

**Problem 3.5.** Four parallel forces of magnitudes 100 N, 150 N, 25 N and 200 N are shown in Fig. 3.14. Determine the magnitude of the resultant and also the distance of the resultant from point A.

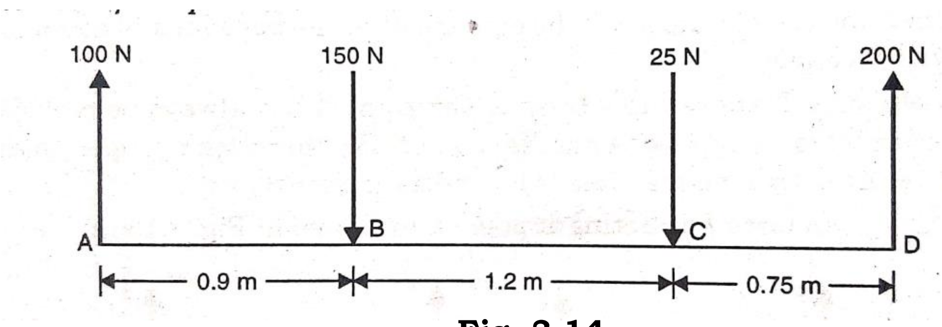



**Fig. 3.14**

**Sol.** Given :

Forces are 100 N, 150 N, 25 N and 200 N.

Distances AB = 0.9 m, BC = 1.2 m, CD = 0.75 m.

As all the forces are acting vertically, hence their resultant R is given by

$R = 100 - 150 - 25 + 200$

(Taking upward force +ve and downward an -ve)

$= 300 - 175 = 125$ N

+ve sign shows that R is acting vertically upwards. To find the distance of R from point A, take the moments of all forces about point A.

Let $x$ = Distance of R from A in metre.

As the force 100 N is passing through A, its moment about A will be zero.

Moment of 150 N force about A = $150 \times AB$

$= 150 \times 0.9$ (clockwise) (-) = -135 Nm

Moment of 25 N force about A = $25 \times AC = 25 \times (0.9 + 1.2)$

$= 25 \times 2.1$ (clockwise) (-) = -52.5 Nm

Moment of 200 N force about A = $200 \times AD$

$= 200 \times (0.9 + 1.2 + 0.75)$

$= 200 \times 2.85$ (anti-clockwise) (+)

= 570 Nm

Algebraic sum of moments of all forces about A

$= -135 - 52.5 + 570 = 382.5$ Nm ...(i)

+ve sign shows that this moment is anti-clockwise. Hence the moment of resultant R about A must be 382.5 Nm, i.e., moment of R should be anti-clockwise about A. The moment of R about A will be anti-clockwise if R is acting upwards and towards the right of A.

Now moment of R about A = $R \times x$. But R = 125

$= 125 \times x$ (anti-clockwise)(+)

$= + 125 \times x$ ...(ii)

Equating (i) and (ii), $382.5 = 125 \times x$ or $\frac{382.5}{125}$ = **3.06 m. Ans.**

∴ Resultant (R = 125 N) will be 125 N upwards and is acting at a distance of 3.06 m to the right of point A.

## 3.6 EFFECT OF A FORCE MOVING PARALLEL TO ITS LINE OF ACTION

A force F acting at a point A, if it is moved to another point B parallel to its line of action, then the original force will be equivalent to the parallel and equal force at B together with a couple.

A given force F applied to a body at any point A can always be replaced by an equal and parallel force applied at another point B together with a couple which will be equivalent to the original force. This is proved as given below.

Let the given force F is acting at point A as shown in Fig. 3.15 (a).

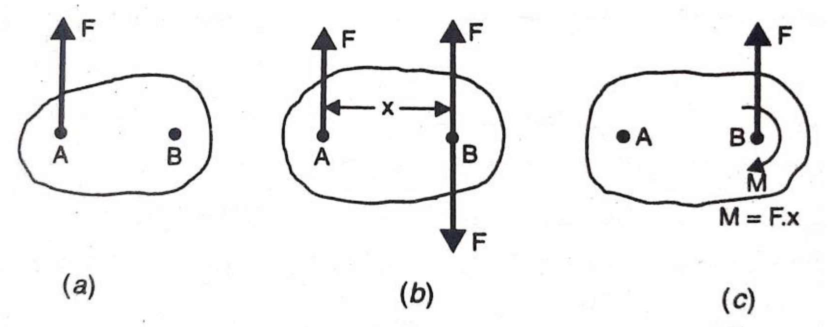



**Fig. 3.15**

This force is to be replaced at the point B. Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A as shown in Fig. 3.15(*b*). The force system of Fig. 3.15(*b*) is equivalent to the single force acting at A of Fig. 3.15(*a*). In Fig. 3.16(*b*) three equal forces are acting. The two forces i.e., force F at A and the oppositely directed force F at B (i.e., vertically downward force at B) from a couple. The moment of this couple is F × $x$ clockwise where $x$ is the perpendicular distance between the lines of action of forces at A and B. The third force is acting at B in the same direction in which the force at A is acting. In Fig. 3.15(c), the couple is shown by curved arrow with symbol M. The force system of Fig. 3.15(c) is equivalent to Fig. 3.15(b). Or in other words the Fig. 3.15(c) is equivalent to Fig. 3,15(a). Hence the given force F acting at A has been replaced by an equal and parallel force applied at point B in the same direction together with a couple of moment F × $x$.

Thus a force acting at a point in a rigid body can be replaced by an equal and parallel force at any other point in the body, and a couple.

**Problem 3.6.** A system of parallel forces are acting on a rigid bar as shown in Fig. 3.16. Reduce this system to :

(i) a single force

(ii) a single force and a couple at A

(iii) a single force and a couple at B.

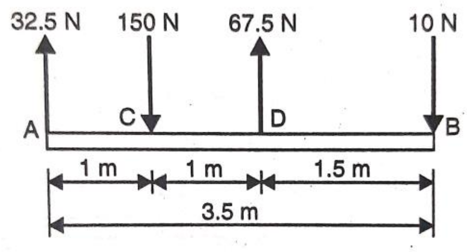



**Fig. 3.16**

**Sol.** Given :

Forces at A, C, D and B are 32.5 N, 150 N, 67.5 N and 10 N respectively.

Distances AC = 1 m, CD = 1 m and BD = 1.5 m.

(i) Single force system. The single force system will consist only resultant force in magnitude and location. All the forces are acting in the vertical direction and hence their resultant (R) in magnitude is given by

$$R = 32.5 - 150 + 67.5 - 10 = \mathbf{-60\ N.\ Ans.}$$

Negative sign shows that resultant is acting vertically downwards.

Let $x$ = Distance of resultant from A towards right. To find the location of the resultant take the moments of all forces about A, we get moment of resultant about A.

= Algebraic sum of moments of all forces about A

or $R \times x = -50 \times AC + 67.5 \times AD - 10\ AB$

(Taking clockwise moment -ve and anticlockwise moment +ve)

or $(-60) \times x = 50 \times 1 + 67.5 \times 2 - 10 \times 3.5 \quad (\because R = -60)$

or $-60x = -150 + 135 - 35 = -50$

$\therefore \quad x = \frac{-50}{-60} = \mathbf{0.833\ m.\ Ans.}$

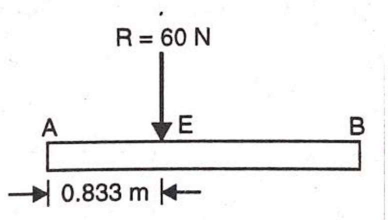



**Fig. 3.16(a)**

Hence the given system of parallel forces is equivalent to a single force 60 N acting vertically downwards at point E at a distance of 0.833 m from A shown in Fig. 3.16(a)

(ii) A single force and a couple at A. The resultant force R acting at point E as shown in Fig. 3.16(a) can be replaced by an equal force applied at point A in the same direction together with a couple. This is shown in Fig. 3.16(c).

The moment of the couple = 60 × 0.839 Nm (clockwise)

= **- 49.98 Nm, Ans.**

(-ve sign is due to clockwise)

(iii) A single force and a couple at B. First find distance BE. But from Fig. 3.16(b), this distance.

BE = AB - AE = 3.5 - 0.833 2.667 m.

The resultant force R acting at point E can be replaced by an equal force applied at point B in the same direction together with a couple.

Hence if the force R = 60 N is moved to the point B, it will be accompanied by a couple of moment 60 × BE or 60 × 2.667 Nm. This is shown in Fig. 3.16(*e*).

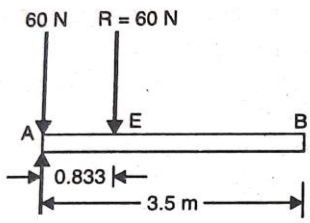



**Fig. 3.16(b)**

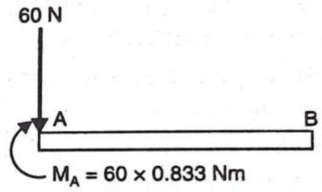



**Fig. 3.16(c)**

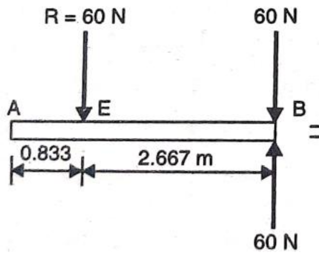



**Fig. 3.16(d)**

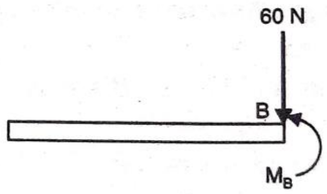



**Fig. 3.16(e)**

The moment of the couple = 60 × 2.667 Nm (anti-clockwise)

= **160 Nm. Ans.**

## 3.7 GENERAL CASE OF PARALLEL FORCES IN A PLANE

Fig. 3.17 shows a number of parallel forces acting on a body in one plane. The forces $F_1$, $F_2$ and $F_4$ are acting in one direction, whereas the forces $F_3$ and $F_5$ are acting in the opposite direction. Let $R_1$ = Resultant of forces

$F_1$, $F_2$ and $F_5$ and $F_2$ = Resultant of forces $F_3$ and $F_5$. The resultant $R_1$ and $R^2$ are acting in opposite direction and are parallel to each other. Now three important cases are possible.

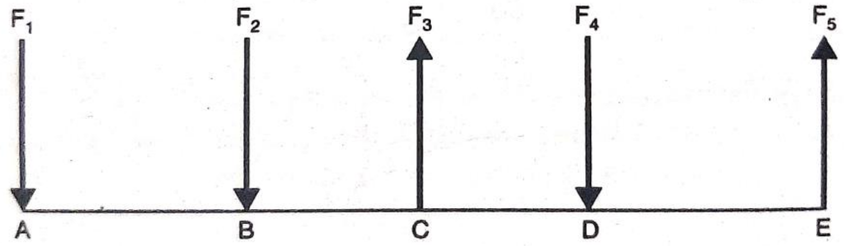



**Fig. 3.17**

1. $R_1$ may not be equal to $R_2$. Then we shall have two unequal parallel forces ($R_1$ and $R_2$) acting in the opposite direction. The resultant R of these two forces ($R_1$ and $R_2$) can he easily obtained. The point of application of resultant R can be obtained by equating the moment of R about any point to the algebraic sum of the moments of individual forces about the same point.

2. $R_1$ is equal to $R_2$. Then we shall have two equal parallel forces ($R_1$ and $R_2$) acting in the opposite direction, 'The resultant R of these two forces will be zero. Now the system may reduce to a couple or the system is in equilibrium. To distinguish between these two cases, the algebraic sum of moments of all forces ($F_1$, $F_2$,..... $F_5$) about any point is taken. If the sum of moments is not zero, the system reduces a resultant couple. The calculated moment gives the moment of this couple.

3. $R_1$ is equal to $R_2$ and eum of moments of all forces ($F_1$, $F_2$, $F_3$, $F_4$, $F_5$,.....) about any point is zero, then the system will not be subjected to any resultant couple but the system will be in equilibrium.

**Problem 3.7**. Determine the resultant of the parallel force system shown in Fig. 3.18

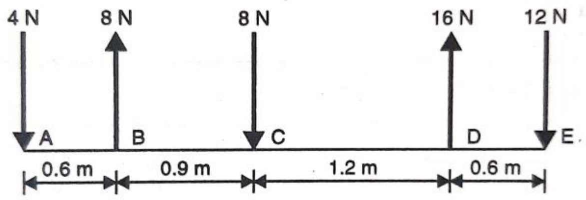



**Fig. 3.18**

**Sol.** Given :

Forces at A, B, C, D and E and 4 N, 8 N, 8 N, 16 N and 12 N respectively.

Distances AB = 0.6 m, BC = 0.9 m,

CD = 1.2 m, and DE = 0.6 m.

Since all the forces are vertical and parallel, hence their resultant is given by

$$R = -4 + 8 - 8 + 16 - 12 = 0$$

As the resultant force on the system is zero, there will be two possibilities. The system has a resultant couple or the system is in equilibrium. To distinguish between these two possibilities, take the sum of moments of all forces about any point. Let us take the moments about point A.

∴ Algebraic sum of moments of all forces about A

$$= 4 \times 0 + 8 \times AB - 8 \times AC + 16 \times AD - 12 \times AE$$

$$= 0 + 8 \times 0.6 - 8 \times (0.6 + 0.9) + 16 \times (0.6 + 0.9 + 1.2) - 12 \times (0.6 + 0,9 + 1.2 + 0.6)$$

$$= 0 + 4.8 - 12 + 16 \times 2.7 - 12 \times 3.3 \text{ Nm}$$

$$= 4.8 - 12 + 43.2 - 39.6 = 48 - 51.6$$

$$= -3.6 \text{ Nm}$$

As the algebraic sum of moments of all forces about any point is not zero, the system will have a resultant couple of magnitude - 3.6 Nm i.e., a clockwise couple. **Ans.**

**Problem 3.8.** Determine the resultant of the parallel forces acting on a body as shown in Fig. 3.19.

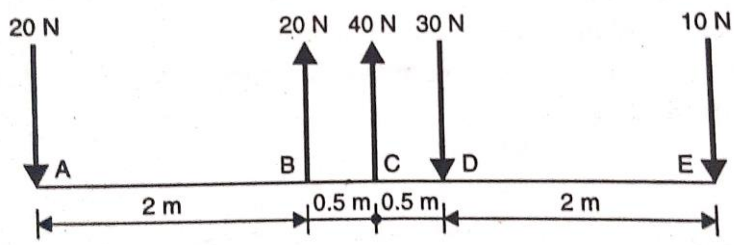



**Fig. 3.19**

**Sol.** Since all the forces are vertical and parallel, hence their resultant is given by

$$R = 20 + 20 + 40 - 30 - 10 = 0$$

Taking moment of all forces about the point. A we get

Resultant moment $= 20 \times 0 + 20 \times 2 + 40 \times 2.5 - 30 \times 3 - 10 \times 5$

$$= 0 + 40 + 100 - 90 - 50 = 140 - 140 = 0$$

As the resultant moment is zero and also the resultant force on the body is zero, the body will be in equilibrium. **Ans.**

## 3.8 EQUIVALENT SYSTEMS

An equivalent system for a given system of coplanar forces, is a combination of a force passing through a given point and a moment about that point. The force is the resultant of all forces acting on the body. And the moment is the sum of all the moments about that point.

Hence equivalent system consists of:

(i) a single force R passing through the given point P and
(ii) a single moment $M_R$

where R = the resultant of all force acting on the body.

$M_R$ = sum of all moments of all the forces about point P.

## 3.9 GENERAL CONDITIONS OF EQUILIBRIUM OF BODIES UNDER COPLANAR FORCES

When some external forces (which may be concurrent or parallel) are acting on a stationary body, the body may start moving or may start rotating about any point. But if the body does not start moving and also does not start rotating about any point, then the body* is said to be in equilibrium.

A stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero and also the algebraic sum of moments of all the external forces about any point in their plane is zero. Mathematically, it is expressed by the equations :

$$\sum F = 0 \qquad \ldots(3.1)$$

$$\sum H = 0 \qquad \ldots(3.2)$$

The sign Σ is known as sigma which is a Greek letter. This sign represents the algebraic sum of forces or moments.

The equation (3.1) is also known as force law of equilibrium whereas the equation (3.2) is known as moment law of equilibrium.

The forces are generally resolved into horizontal and vertical components. Hence equation (3.1) is written as

$$\sum F_x = 0 \quad \ldots(3.3)$$

and $$\sum F_y = 0 \quad \ldots(3.4)$$

where $\sum F_x$ = Algebraic sum of all horizontal components

and $\sum F_y$ = Algebraic sum of all vertical components.

**3.9.1. Equations of Equilibrium for Non-concurrent Forces Systems.** A non-concurrent force systems will be in equilibrium if the resultant of all forces and moment is zero.

Hence the equations of equilibrium are

$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

**3.9.2. Equations of Equilibrium for Concurrent Force System.** For the concurrent forces, the lines of action of all forces meet at a point, and hence the moment of those force about that very point will be zero or $\sum M = 0$automatically.

Thus for concurrent force system, the condition $\sum M = 0$ becomes redundant and only two conditions, i.e., $\sum F_x = 0$ and $\sum F_y = 0$ are required.

**3.9.3. Force Law of Equilibrium.** Force law of equilibrium is given by equation (3.1) or by equations (3.3) and (3.4). Let us apply this law to the following important force system :

(i) Two force system
(ii) Three force system
(iii) Four force system.

**3.9.4. Two Force System.** When a body is subjected to two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite as shown in Fig. 3.20.

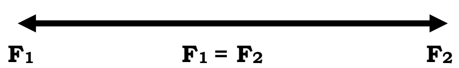



**Fig. 3.20**

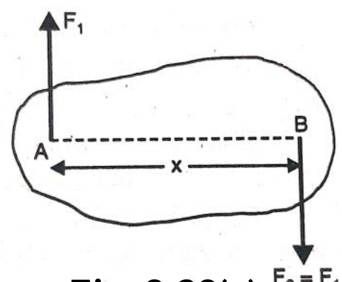



**Fig. 3.20(a)**

If the two forces acting on a body are equal and opposite but are parallel, as shown in Fig. 3.20(a), then the body will not be in equilibrium. This is due to the fact that the three conditions of equilibrium will not be satisfied. This is proved as given below:

(i) Here $\sum F_x = 0$ as there is no horizontal force acting on the body. Hence first condition of equilibrium is satisfied.

(ii) Also here $\sum F_y = 0$ as $F_1 = F_2$.

Hence second condition of equilibrium is also satisfied:

(iii) $\sum M$ about any point should be zero. The resultant moment about point A is given by

$$M_A = - F_2 \times AB \qquad \text{(- ve sign is due to clockwise moment)}$$

But $M_A$ is not equal to zero. Hence the third condition is not satisfied.

Hence a body will not be in equilibrium under the action of two equal and opposite parallel forces.

Two equal and opposite parallel forces produce a couple and moment of the couple is $- F_1 \times AB$ (See Fig. 3.20(a)).

**3.9.5. Three Force System.** The three forces acting on a body which is in equilibrium may be either concurrent or parallel. Let us first consider that the body is in equilibrium when three forces, acting on the body, are concurrent. This is shown in Fig. 3.21.

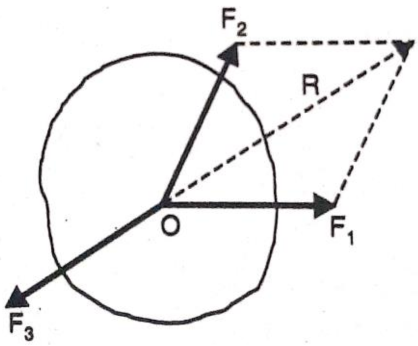



**Fig. 3.21**

(a) When three forces are concurrent. The three con- current forces $F_1$, $F_2$ and $F_3$ are acting on a body at point O and the body is in equilibrium. The resultant of $F_1$ and $F_2$ is given by R. If the force $F_3$ is collinear, equal and opposite to the resultant R, then the body will be in equilibrium. The force $F_3$ which is equal and opposite to the resultant R is known as equilibrant. Hence for three concurrent forces acting on a body when the body is in equilibrium, the resultant of the two forces should be equal and opposite to the third force.

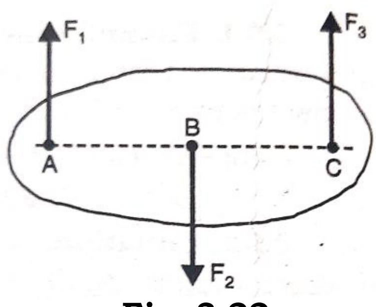



**Fig. 3.22**

(b) When three forces are parallel. Fig. 3.22 shows a body on which three parallel forces $F_1$, $F_2$, and $F_3$ are acting and the body is in equilibrium. If three forces $F_1$, $F_2$, and $F_3$ are acting in the same direction, then there will be a resultant R = $F_1$ + $F_2$, + $F_3$ and body will not be in equilibrium. The three forces are acting in opposite direction and their magnitude is so adjusted that there is no resultant force and body is in equilibrium. Let us suppose that F, is acting in opposite direction as shown in Fig. 3.22.

Now let us apply the three conditions of equilibrium :

(i) $\sum F_x = 0$ as there is no horizontal force acting on the body
(ii) $\sum F_y = 0$ i.e., $F_1 + F_3 = F_2$
(iii) $\sum M = 0$ about any point.

Taking the moments of $F_1$, $F_2$ and $F_3$ about point A,

$$\sum M_A - F_2 \times AB + F_3 \times AC$$

(Moment of $F_3$ is anti-clockwise whereas moment of $F_2$ is clockwise)

For equilibrium, $\sum M_A$ should be zero

i.e., $$-F_2 \times AB + F_3 \times AC = 0$$

If the distances AB and AC are such that the above equation is satisfied, then the body will be in equilibrium under the action of three parallel forces.

**3.9.6. Four Force System**. The body will be in equilibrium if the resultant force in horizontal direction is zero (i.e., $\sum F_x = 0$), resultant force in vertical direction is zero (i.e., $\sum F_y = 0$) and moment of all forces about any point in the plane of forces is zero (i.e., $\sum M = 0$).

**Problem 3.9.** Two forces $F_1$ and $F_2$ are acting on a body and the body is in equilibrium. If the magnitude of the force $F_1$ is 100 N and its acting at O long $x$-axis as shown in Fig. 3.23, then determine the magnitude and direction of force $F_2$.

**Sol.** Given :

Force, $F_1 = 100$ N

The body is in equilibrium under the action of two forces $F_1$ and $F_2$.

When two forces are acting on a body and the body is in equilibrium, then the two forces should be collinear, equal and opposite.

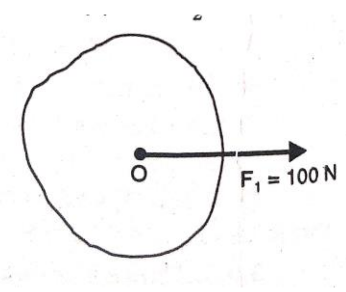



**Fig. 3.23**

∴ $F_2 = F_1 = 100$ N

The force $F_2$ should pass though O, and would be acting in the opposite direction of $F_1$.

**Problem 3.10.** Three forces $F_1$, $F_2$ and $F_3$ are acting on a body as shown in Fig. 3.24 and the body is in equilibrium. If the magnitude of force F3 is 400 N, find the magnitude of force $F_1$ and $F_2$.

**Sol.** Given:

Force $F_3 = 400$ N.

As the body is in equilibrium, the resultant force in x-direction should be zero and also the resultant force in y-direction should be zero.

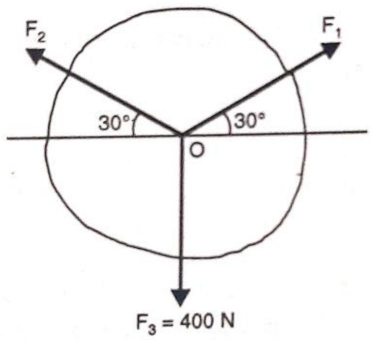



**Fig. 3.24**

(i) For $\sum F_x = 0$, we get

$F_1 \cos 30° - F_2 \cos 30° = 0$

or $F_1 - F_2 = 0$

or $F_1 = F_2$ ...(i)

(ii) For $\sum F_y = 0$, we get

$F_1 \sin 30° + F_2 \sin 30° - 400 = 0$

or $F_1 \times 0.5 + F_2 \times 0.5 = 400$

or $F_1 \times 0.5 + F_2 \times 0.5 = 400$ $(\because F_2 = F_1)$

or $F_1 =$ **400 N. Ans.**

Also $F_2 = F_1 =$ **400 N. Ans.**

**2nd Method**

If three forces are acting on a body at a point and the body is in equilibrium, Lami's Theorem can be applied.

Using Lami's theorem

$$\frac{F_1}{\sin 120°} = \frac{F_2}{\sin 120°} = \frac{400}{\sin 120°}$$

or $F_1 = F_2 =$ **400 N Ans.**

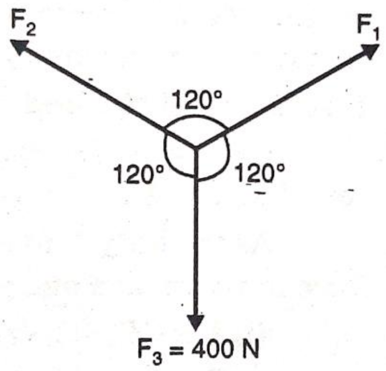



**Fig. 3.25**

**Problem 3.11.** Three parallel forces $F_1$, $F_2$ and $F_3$ are acting on a body as shown in Fig. 3.26 and the body is in equilibrium. If force $F_1 = 250$ N and

$F_3$ = 1000 N and the distance between $F_1$ and $F_2$ = 1.0 m, then determine the magnitude of force $F_2$ and the distance of $F_2$ from force $F_3$.

**Sol.** Given :

Force $F_1 = 250$ N

Force $F_3 = 1000$ N

Distance AB = 1.0 m

The body is in equilibrium.

Find $F_2$ and distance BC.

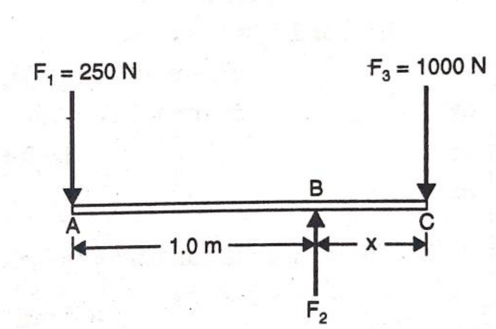



**Fig. 3.26**

For the equilibrium of the body, the resultant force in the vertical direction should be zero (here there is no force in horizontal direction).

$\therefore$ For $\sum F_y = 0$, we get

$$F_1 + F_3 - F_2 = 0$$

or $250 + 1000 - F_2 = 0$

or $F_2 = 250 + 1000 =$ **1250 N. Ans.**

For the equilibrium of the body, the moment of all forces about any point must be zero.

Taking moments of all forces about point A and considering distance BC = $x$, we get

$$F_2 \times AB - AC \times F_3 = 0$$

or $1250 \times 1.0 - (1 + x) \times 1000 = 0$ $\quad (\because AC = AB + BC = 1 + x)$

or $1250 - 1000 - 1000x = 0$

or $250 = 1000x$

or $x = \frac{250}{1000} =$ **0.25m. Ans.**

**Problem 3.12.** The five forces $F_1$, $F_2$, $F_3$, $F_4$ and $F_5$ are acting at a point on a body as shown in Fig. 3.27 and the body is in equilibrium. If $F_1$ = 18 N, $F_2$ = 22.5 N, $F_3$ = 15 N and $F_4$ = 30 N, find the force $F_5$ in magnitude and direction.

**Sol.** Given:

Forces, $F_1 = 18$ N, $F_2 = 2.25$ N,

$F_3 = 16$ N and $F_4 = 30$ N.

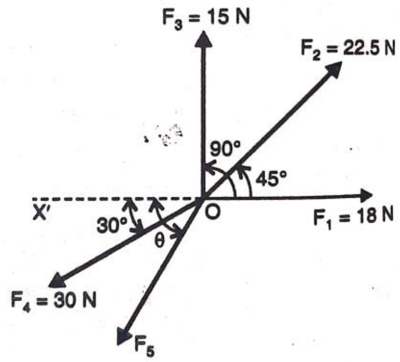



**Fig. 3.27**

The body is in equilibrium. Find force $F_5$ in magnitude and direction. This problem can be solved analytically and graphically.

**1. Analytical Method**

Let θ = Angle made by force $F_5$ with horizontal axis 0-X.

As the body is in equilibrium, the resultant force in x-direction and y-direction should be zero.

(i) For $\sum F_x = 0$, we get

$$F_1 + F_2 \cos 45° - F_4 \cos 30° - F_5 \cos \theta = 0$$

or $$18 + 22.5 \times 0.707 - 30 \times 0.866 - F_5 \cos \theta = 0$$

or $$18 + 15.9 - 25.98 - F_5 \cos \theta = 0$$

or $$F_5 \cos \theta = 18 + 15.9 - 25.98$$

or $$F_5 \cos \theta = 7.92 \quad \text{...(i)}$$

(ii) For $\sum F_y = 0$, we get

$$F_2 \sin 45° + F_3 - F_4 \sin 30° - F_5 \sin \theta = 0$$

or $$22.5 \times 0.707 + 15 - 30 \times 0.5 - F_5 \sin \theta = 0$$

or $$15.9 + 15 - 16 - F_5 \sin \theta = 0$$

or $$F_5 \sin \theta = 15.9 \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we get $\frac{F_5 \sin \theta}{F_5 \sin \theta} = \frac{15.9}{7.92}$

or $\tan \theta = 2.0075$

∴ $\theta = \tan^{-1} 2.0075 =$ **63.52**°. **Ans.**

Substituting the value of 0 in equation (i), we get

$$F_5 \cos 63.52° = 7.92$$

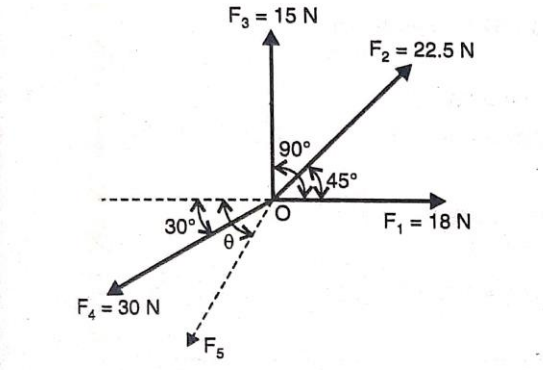
∴ $F_5 = \frac{7.92}{\cos 63.52} =$ **17.76 N. Ans.**

**2. Graphical Method**

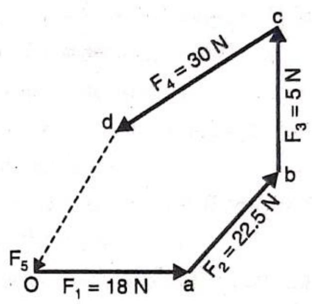
(i) First draw a space diagram with given four forces $F_1$, $F_2$, $F_3$ and $F_4$. at correct angles as shown in Fig. 3.28 (*a*).

(ii) Now choos6 a suitable scale, day 1 cm = 5 N for drawing a force diagrams. Take any point O in the force diagram as shown in Fig. 3.28 (b).

(iii) Draw line *Oa* parallel to force $F_1$ and cut Oa = $F_1$ = 18 N to the same scale.




**(a) Space diagram**




**(b) Force diagram**

**Fig. 3.28**

(iv) From *a*, draw the line *ab* parallel to $F_2$ and cut *ab* = $F_2$ = 22.5 N

(v) From *b*, draw the line *bc* parallel to $F_3$ and cut *bc* = $F_3$ = 15 N

(vi) From *c*, draw the line *cd* parallel to $F_4$ and cut *cd* = $F_4$ = 30 N

(vii) Now join *d* to *O*. Then the closing side do represents the force $F_5$ in magnitude and direction. Now measure the length do.

By measurement, length *dO* = 3.55 cm.

∴ Force $F_5$ = Length do Scale = 3.55 × 5 = **17.75 N. Ans.**

The direction is obtained in the space diagram by drawing the force $F_5$ parallel to line *dO*.

Measure the angle θ, which is equal to 63.5°. Or the force $F_5$ is making an angle of 180 + 63.5 = 243.5° with the force $F_1$.

**Problem 3.13.** Fig. 3.28(c) shows the coplanar system of forces acting on a flat plate. Determine : (i) the resultant and (ii) x and y intercepts of the resultant.

**Sol.** Given :

Force at A = 2240 N

Angled with *x*-axis = 63.43°

Force at B = 1805 N

Angled with *x*-axis = 33.67°

Forced at C = 1500 N

Angled with *x*-axis = 60°

Lengths OA = 4 m

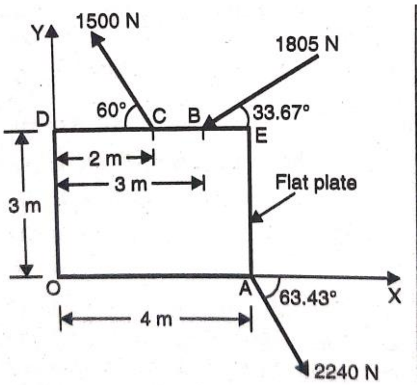



**Fig. 3.28(c)**

DB = 3 m

DC = 2 m

OD = 3 m.

Each force is resolved into X and Y components as shown in Fig. 3.28 (d).

(i) Force at A = 2240 N.

Its X-component = 2240 × cos 63.43° = 1001.9 N

Its Y-component = 2240 × sin 63.43° = 2003.4 N

(ii) Force at B = 1805 N.

Its X - component = 1805 × cos 33.67° = 1502.2 N

Its Y-component = 1805 × sin 33.67° = 1000.7 N

(iii) Force at C= 1500 N.

Its X-component = 1500 × cos 60° = 750 N

Its Y-component = 1500 x sin 60° = 1299 N

The net force along X-axis,

$$R_x = \text{F}\sum \text{F}_x = 1001.9 - 1502.2 - 750 = -1250.3 \text{ N}$$

The resultant force is given by,

$$R_y = \text{F}\sum \text{F}_y = -2003.4 - 1000.7 + 1299 = -1705.1 \text{ N}$$

(i) The resultant force is given by,

$$\text{R} = \sqrt{\text{R}_x{}^2 + \text{R}_x{}^2} = \sqrt{(-1250.3)^2 + (-1705.1)^2}$$

$$= \sqrt{1563250 + 2907366} = \mathbf{2114.4\ N.\ Ans.}$$

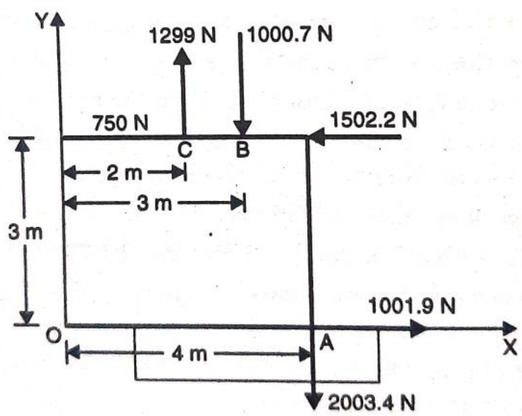



**Fig. 3.28(d)**

The angle made by the resultant with $x$-axis is given by

$$\tan\theta = \frac{R_y}{R_x} = \frac{-1705.1}{-1250.3} = 1.363$$

$\therefore$ $\theta = \tan^{-1} 1.363 = 53.70$

The net moment* about point $O$,

$M_O. = 2003.4 \times 4 + 1000.7 \times 3 - 1299 \times 2 - 1502.2\ 3 - 750 \times 3$

$= 8012.16 + 3002.1 - 2598 - 4506.6 - 2250$

$= 11014.26 - 9364.6 = 1659.55$ Nm

As the net moment about O is clockwise, hence the resultant must act towards right of origin O, making an angle = 53.7° with $x$-axis as shown in Fig. 3.28(e). The components $R_x$ and $R_y$ are also negative. Hence this condition is also satisfied.

(ii) Intercepts of resultant on x-axis and y-axis (Refer to Fig. 3.28(e)).

Let $x$ = Intercept of resultant along $x$-axis.

$y$ = Intercept of resultant along y-axis.

The moment of a force about a point is equal to the sum of the moments of the components of the force about the same point. Resolving the resultant (R) into its component, $R_x$ and $R_y$ at F.

Moment of R about O = Sum of moments of $R_x$ and $R_y$ at O

But moment of R about O

$= 1659.66$ ($M_O = 1659.66$)

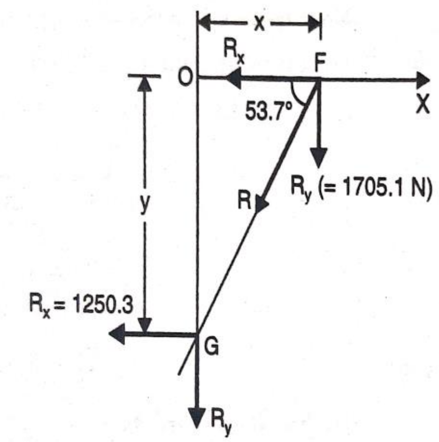



**Fig. 3.28(e)**

$\therefore$ $1659.66 = R_x \times O + R_y \times x$

(as $R_x$ at F passes through O hence it has no moment)

$\therefore$ $1659.66 = 1705.1 \times x$ ($\therefore R_y = 1705.1$)

$\therefore$ $x = \frac{1659.66}{1705.10}$ = **0.97 m right of O. Ans.**

To find y-intercept, resolve the resultant R at G into its component $R_x$ and $R_y$.

$\therefore$ Moment of R about O = Sum of moments of $R_x$ and $R_y$ at O

or $1659.66 = R_x \times y + R_y \times O.$

(At G, $R_y$ passes through O and hence has no moment)

$\therefore \quad 1659.66 = 1250.3 \times y$

$\therefore \quad y = \frac{1659.66}{1250.30}$ = **1.32 m below of O. Ans.**

**Problem 3.14.** A lamp weighing 5 N is suspended from the ceiling by a chain. It is pulled aside by a horizontal cord until the chain makes an angle of 60° with the ceiling as shown in Fig. 3.29. Find the tensions in the chain and the cord by applying Lami's theorem and also by graphical method.

**Sol.** Given :

Weight of lamp = 5 N

Angle made by chain with ceiling = 60°

Cord is horizontal as shown in Fig. 3.29.

(i) By Lami's theorem

Let $T_1$ = Tension (or pull) in the cord

$T_2$ = Tension (or pull) in the chain.

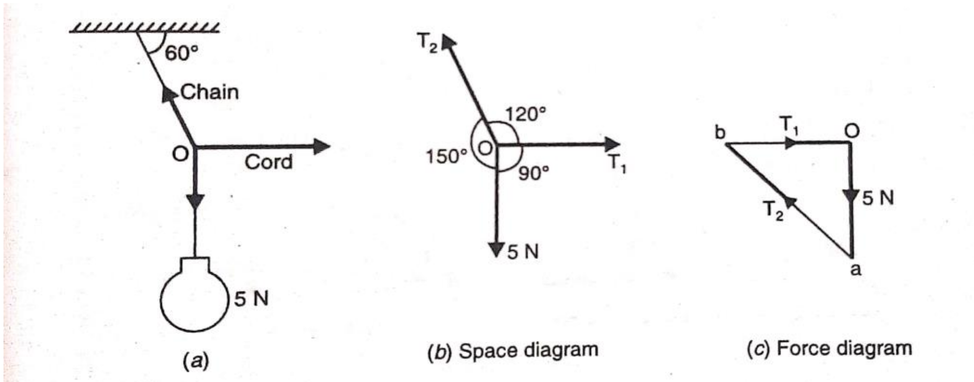



**Fig. 3.29**

Now from the geometry, it is obvious that angles between T, and lamp will be 90°, between lamp and $T_2$ 150° and between $T_2$ and $T_1$ 120°.

[Refers to Fig. 3.29(b)]

Applying Lami's theorem, we get

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 90°} = \frac{5}{\sin 120°}$$

$\therefore \quad T_1 = 5 \times \frac{\sin 150°}{\sin 120°}$ = **2.887 N. Ans.**

and $\quad T_2 = 5 \times \frac{\sin 90°}{\sin 120°}$ = **5.774 N. Ans.**

(ii) By Graphical method

(1) First draw the space diagram at correct angles as shown in Fig. 3.29(b). Now choose a suitable scale say 1 cm = 1 N for drawing a force diagram as shown in Fig. 3.29(c). Take any point in the force diagram.

(2) From O, draw the line O*a* vertically downward to represent the weight of the lamp. Cut O*a* = 5 N.

(3) From a, draw the line *ab* parallel to $T_2$. The magnitude of $T_2$ is unknown. Now from O, draw the line O*b* horizontally (i.e., parallel to $T_1$) cutting the line *ab* at point *b*.

(4) Now measure the lengths *ab* and *b*0.

Then *ab* represents $T_2$ and *b*O represents $T_1$. By measurements, *ab* = 5.77 cm and *b*O= 2.9 cm.

∴ Pull in the cord = *b*O = 2.9 cm × scale = 2.9 × 1

= **2.9 N. Ans.**

Pull in the chain = *ab* = 5.77 cm × scale = 5.77 × 1

= **5.77 N. Ans.**

**Problem 3.15.** On a horizontal line PQRS 12 cm long, where PQ = OR = RS = 4cm, forces of 1000 N, 1500 N, 1000 N and 500 Nare acting at P, Q, R and S respectively, all downwards, their lines of action making angles of 90°, 60°, 45° and 30° respectively with PS. Obtain the resultant of the system completely in magnitude, direction and position graphically and check the answer analytically.

**Sol.** Given :

PQ = QR = RS = 4 cm

Force at P = 1000 N. Angle with PS = 90°

Force at Q = 1500 N. Angle with QS = 60°

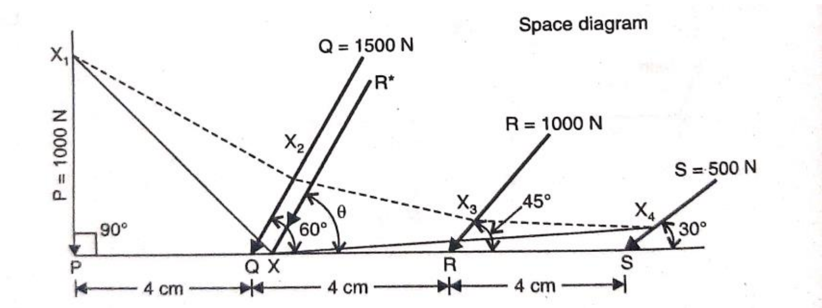



**Fig. 3.30(a)**

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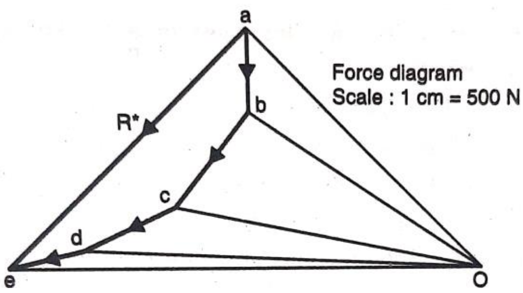


**Fig. 3.30(b)**

Force at R = 1000 N. Angle with RS = 45°
Force at S = 500 N. Angle with PS = 30°

**Graphical method**

Draw the space diagram of the forces as shown in Fig. 3.30(a). The procedure is as follows:

(i) Draw a horizontal line PQRS = 12 cm in which take PQ = QR = RS = 4 cm.

(ii) Draw the line of action of forces P, Q, R, S of magnitude 1000 N, 1500 N, 1000 N and 500 N respectively at an angle of 90°, 60°, 45° and 30° respectively with line PS as shown in Fig. 3.30 (*a*).

**Magnitude and Direction of Resultant Force (R*)**

To find the magnitude and direction of the resultant force, the force diagram is drawn as shown in Fig. 3.30(a) as given below:

(i) Draw the vector *ab* to represent the force 1000 N to a scale of 1 cm = 500 N. The vector ab is parallel to the line of action of force P.

(ii) From point b, draw vector, *bc* = 1500 N and parallel to the line of action of force Q. Similarly the vectors, *cd* = 1000 N and parallel to line of action of force R and *de* = 500 N and parallel to the line of action of force S, are drawn.

(iii) Join *ae* which gives the magnitude of the resultant. Measuring *ae*, the resultant force is equal to 3770 N.

(iv) To get the line of action of the resultant, choose any point on force diagram (called the pole) and join O*a*, O*b*, O*c*, O*d* and O*e*.

(v) Now choose any point $X_1$ on the line of action of force P and draw a line parallel to O*a*.

(vi) Also from the point $X_1$ draw another line parallel to O*b*, which cuts the line of action of force Q at $X_2$ Similarly from point $X_2$, draw a line parallel to O$c$ to cut the line of action of force R at $X_3$. From point $X_3$, draw a line parallel to O$d$ to cut the line of action of force S at $X_4$:

(vii) From point $X_4$, draw a line parallel O$e$.

(viii) Produce the first line (i.e., the line from $X_1$ and parallel to O$a$) and the last line (i.e., the line from $X_4$ and parallel to O$e$) to interest at X. Then the resultant must pass through this point.

(ix) From point X, draw a line parallel to $ae$ which determines the line of action of resultant force. measure PX. By measurements:

Resultant force, $R^* = 3770$ N

Point of action, PX = 4.20 cm

Direction, $\theta = 60° \, 30'$ with PS.

**Analytical method**

In analytical method, all the forces acting can be resolved horizontally and vertically. Resultant of all vertical and horizontal forces can be calculated separately and then the final resultant can be obtained.

Resolving all forces and considering the system for vertical forces only.

Vertical force at P = 1000 N

Vertical force at Q = 1500 sin 60° = 1299 N

Vertical force at R = 1000 sin 45° = 707 N

Vertical force at S = 500 sin 30° = 250 N

The vertical forces are shown in Fig. 3.31.

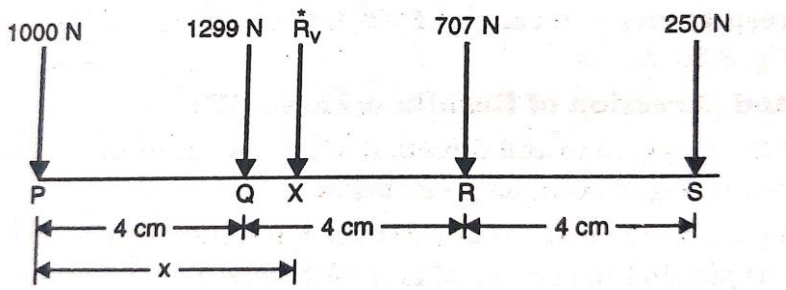


**Fig. 3.31**

Let $R_y$ = the resultant of all vertical forces and acting at a distance $x$ cm from P.

$$= 1000 + 1299 + 707 + 250 = 3256 \text{ N}$$

Taking moments of all vertical forces about point P,

$$R_V^* \times x = 1299 \times 4 + 707 \times 8 + 250 \; 12 = 13852$$

∴ $$x = \frac{13852}{R_Y^*} = \frac{13852}{3256}\ \ 4.25 \text{ cm}$$

Now consider the system for horizontal forces only,

Horizontal force at P = 0

Horizontal force at Q = 1500 × cos 60º = 750 N

Horizontal force at R = 1000 × cos 45º = 707 N

Horizontal force at S = 500 × cos 30º = 433 N

Resultant of all horizontal forces will be,

$$R_H{}^* = 0 + 750 + 707 + 433 = 1890 \text{ N}$$

The resultant R* of $R_V{}^*$ and $R_H{}^*$ will also through point X which is at a distance of 4.25 cm from P.

∴ R* = $\sqrt{R_V{}^* + R_H{}^*} = \sqrt{3256^2 + 1890^2}$ = **3764 N. Ans.**

The resultant will make an angle θ with PS and is given by

$$\tan\theta = \frac{R_V{}^*}{R_H{}^*} = \frac{3256}{1890} = 1.723$$

∴ θ = $\tan^{-1}$ 1.723 = 59.9*

Then the resultant of 3764 N makes and angle 59.9* with PS and passing through point X which is at a distance of 4.25 cm from point P.

The result confirms closely with the values obtained by graphical method.

## STUDENT ACTIVITY

**1.** Explain the terms : Momentum of a body.

**2.** Define and explain the Newton Laws of motion for linear motion.

## SUMMARY

1. Parallel forces are having their lines of action parallel to each other.
2. The moment of a force about any point is the product of force and perpendicular distance between the point and line of action of force.
3. Anti-clockwise moment is taken +ve whereas clockwise moment is taken -ve.
4. Varignon's principle states that the moment of a force about any point is equal to the algebraic sum of moments of its components about that point.
5. Like parallel forces are parallel to each other and are acting in the same direction, whereas the unlike parallel forces are acting in opposite direction.
6. The resultant of two like parallel forces is the sum of the two forces and acts at a point between the line in such a way that the resultant divides the distance in the ratio inversely proportional to the magnitudes of the forces.
7. When two equal and opposite parallel forces act on a body at some distance apart, the two forces form a couple which has a tendency to rotate the body. The moment of this couple is the product of either one of the forces and perpendicular distance between the forces.
8. A given force F applied to a body at any point A can always be replaced by an equal force applied at another point B in the same direction together with a couple.
9. If the resultant of a number of parallel forces is not zero, the system can be reduced to a single force, whose magnitude is equal to the algebraic sum of all forces. The point of application of this single force is obtained by equating the moment of this single force about any point to the algebraic sum of moments of all forces acting on the system about the same point.
10. If the resultant of a number of parallel forces is zero, then the system may have a resultant couple or may be in equilibrium. If the algebraic sum of moments of all forces about any point is not zero, then system will have a resultant couple. But if the algebraic sum of moments of all forces about any point is zero, the system will be in equilibrium.
11. The principle of equilibrium states that a stationary body will be in equilibrium if the algebraic sum of all the forces is zero and also the algebraic sum of moments of all the external forces is zero.
12. The conditions of equilibrium are written mathematically as $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M = 0$. The sign is known as sigma and this sign represents the algebraic sum.
13. When a body is subjected to two forces, the body will be in equilibrium if the two forces are collinear, equal and opposite.

14. Two equal and opposite parallel forces produces a couple whose moment is equal to either force multiplied by their perpendicular distance.
15. If three concurrent forces are acting on a body and the body is in equilibrium, then the resultant of two forces should be equal and opposite to the third force.
16. Free body diagram of a body is a diagram in which the body is completely isolated from its support and the supports are replaced by the reactions which these supports exert on the body.

## TEST YOURSELF

**(A) Theoretical Problem**

1. Define the terms: Coplanar parallel forces, like parallel forces and unlike parallel forces.
2. Define and explain the moment of a force. Differentiate between clockwise moment and anti-clockwise moment.
3. (a) State the Varignon's principle. Also give the proof of Varignon's principle.
   (b) Differentiate between :
      (i) Concurrent and non-concurrent forces,
      (ii) Coplanar and non-coplanar forces,
      (iii) Moment of a force and couple.
4. Define moment of a force about a point and show that the algebraic sum of the moments of two coplanar forces about a point is equal to the moment of their resultant about that point.
5. What are the different types of parallel forces ? Distinguish between like and unlike parallel forces ?
6. Prove that the resultant of two like parallel forces $F_1$ and $F_2$ is $F_1 + F_2$. prove that the resultant divides the line of joining the points of action of $F_1$ and $F_2$ internally in the inverse ratio of the forces.
7. Prove that in case of two unlike parallel forces the resultant lies outside the line joining the points of action of the two forces and on the same side as the larger force.
8. Describe the method of finding the line of action of the resultant of a system of parallel forces.
9. The resultant of a system of parallel forces is zero, what does it signify ?
10. Describe the method of finding the resultant of two unlike parallel forces which are equal in magnitude.
11. Prove that a given force F applied to a body at any Point A can always be replaced by an equal force applied at another point B together with a couple.
12. State the principle of moment.
13. Indicate whether the following statements are True or False.
    (i) Force is an agency which tends to cause motion.

(ii) The tension member of a frame work is called a street.
(iii) The value of g reduces slightly as we move from poles towards the equator.
(iv) Coplanar forces arc those which have the same magnitude and direction.
(v) A couple consists of two unequal and parallel forces acting on a body, having the same line of action.
(vi) A vector diagram of a force represents its magnitude, direction, sense and point of application.
(vii) The force of gravitation on a body is called its weight.
(viii) The cenre of gravity of a body is the point, thought which is resultant of parallel forces passes is whatever position may the body by placed.

[**Ans.** (i) True, (ii) False, (iii) True, (iv) False, (v) False, (vi) False, (vii) True, (viii) True)

**14.** Define and explain the terms. Principle of equilibrium, force law of equilibrium and moment law of equilibrium.
**15.** A number of forces are acting on a body. What are conditions of equilibrium, so that the body is in equilibrium ?
**16.** Two forces are acting on a body and the body is in equilibrium. What conditions should be fulfilled by these two forces ?
**17.** How will you prove that a body will not be in equilibrium when the body is subjected to two forces which are equal and opposite but are parallel ?
**18.** Explain the statement "Two equal and opposite parallel forces produces a couple".
**19.**
(a) What conditions must be fulfilled by a set of three parallel forces which are acting on a body and body is in equilibrium ?
(b) State the graphical conditions that must be satisfied for the equilibrium of a system of coplanar forces.

**(B) Numerical Problems**

**1.** Four forces of magnitudes 20 N, 40 N, 60 N and 80 N are acting respectively along the four sides of a square ABCD as shown in Fig. 3.32. Determine the resultant moment about point A.

Each side of square is 2 m. [**Ans.** 200 Nm anti-clockwise]

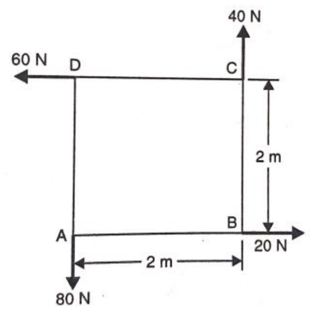


**Fig. 3.32**

**2.** A force of 50 N is acting at a point A as shown in Fig. 3.33. Determine the moment of this force about O.

[**Ans.** 100 Nm clockwise]

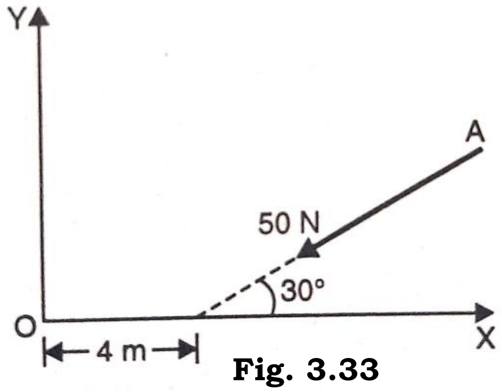


**Fig. 3.33**

**3.** Three like parallel forces 20 N, 40 N and 60 N are acting at points A, B and C respectively on a straight line ABC. The distances are AB = 3 m and BC = 4 m.

Find the resultant and also the distance of the resultant from point A on line ABC. [**Ans**. 120 N, 4.5 m]

**4.** The three like parallel forces 101 N, F and 300 N are acting as shown in Fig. 3.34. If the resultant R = 600 N and is acting at a distance of 45 cm from A then find the magnitude of force F and distance of F and A.

[**Ans.** 200 N, 30 cm]

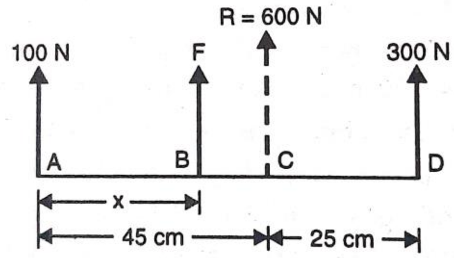


**Fig. 3.34**

**5.** Four parallel forces of magnitudes 100 N, 200 N, 50 N and 400 N are shown in Fig. 3.35. Determine the magnitude of the resultant and also the distance of the resultant from point A.

[**Ans**. R = 350 N, 3.07 m]

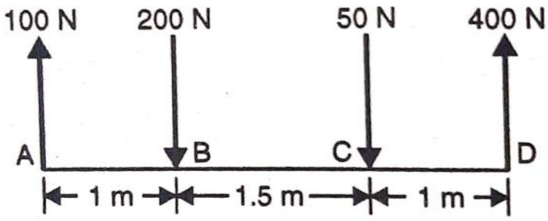


**Fig. 3.35**

**6.** A system of parallel forces are acting on a rigid bar as shown in Fig. 3.36, Reduce this system to :

(i) a single force

(ii) a single force and a couple at A

(iii) a single force and a couple at B.

**Ans.** (i) R = 120 N at 2.83 m from A
(ii) R = 120 N and $M_A$ = - 340 Nm
(iii) R = 120 N and $M_B$ = 120 Nm)

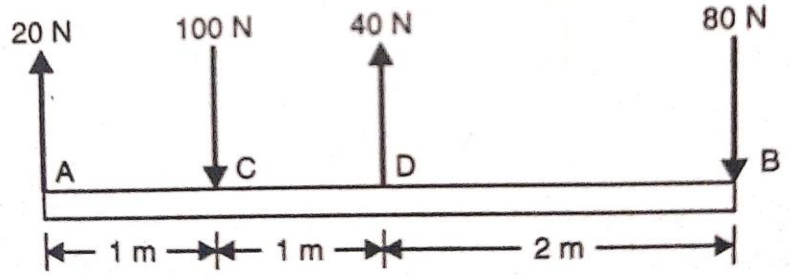


**Fig. 3.36**

**7.** Five forces are acting on a body as shown in Fig. 3.37. Determine the resultant.

[**Ans.** R = 0, Resultant couple = 10 Nm]

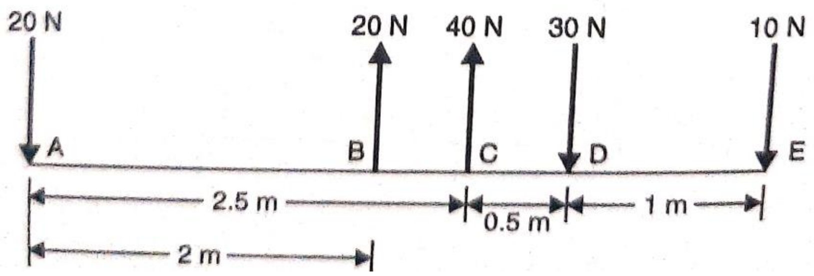


**Fig. 3.37**

**8.** Determine the resultant of the parallel forces shown in Fig. 3.38.

[**Ans.** Body is in equilibrium]

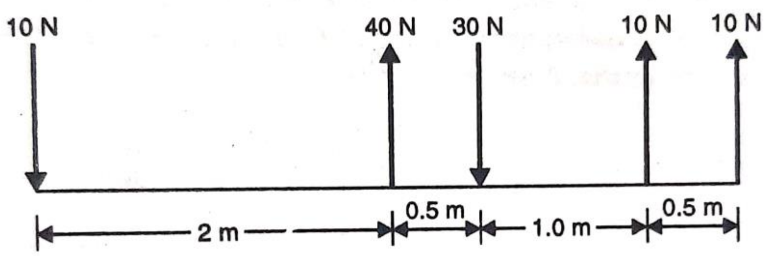


**Fig. 3.38**

**9.** Three forces $F_1$, $F_2$ and $F_3$ are acting on a body as shown in Fig. 3.39 and the body is in equilibrium. If the magnitude of force $F_3$ is 250 N, find the magnitudes of force $F_1$ and $F_2$.

[**Ans.** F = 125 N and F2 = 215.6 N]

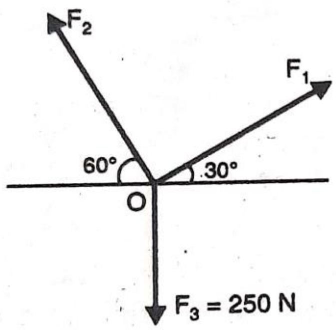


**Fig. 3.39**

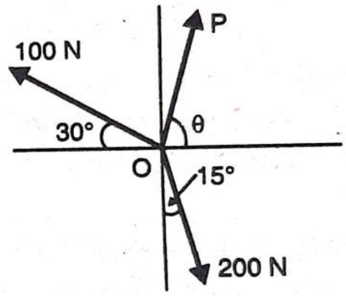


**Fig. 3.40**

**10.** Three forces of magnitudes P, 100 N and 200 N are acting at a point O as shown in Fig. 3.40. Determine the magnitude and direction of the force P.

[**Ans.** P = 147 N and 0 = 76.8°]

**11.** Three parallel forces $F_1$, $F_2$ and $F_3$ are acting on a body as shown in Fig. 3.41 and the body is in equilibrium. If force $F_1$ = 300 N and $F_3$ = 1000 N and the distance between $F_1$ and $F_2$ = 2.0 m, then determine the magnitude of force $F_2$ and distance of $F_3$ from force $F_2$.

[**Ans.** 1300 N, 0.6 m]

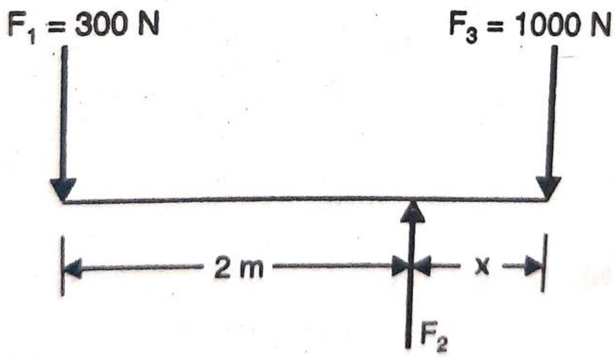


**Fig. 3.41**

**12.** Three forces of magnitude 40 kN, 15 kN and 20 kN are acting at a point O. The angles made by 40 kN, 15 kN and 20 kN forces with x-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force,

[**Ans.** 30,41 kN and 85,28° with x-axis]

**13.** A lamp weighing 10 Nis suspended from the ceiling by a chain. It is pulled aside by a horizontal cord until the chain makes an angle of 60°

with the ceiling. Find the tensions in the chain and the cord by applying Lami's theorem and also by graphical method.

[**Ans.** 11.54 N and 5.77 N]

**14.** Draw the free-body diagram of a ball of weight W supported by a string AB and resting on a smooth horizontal surface at C when a horizontal force is applied to the ball as shown in Fig. 3.42.

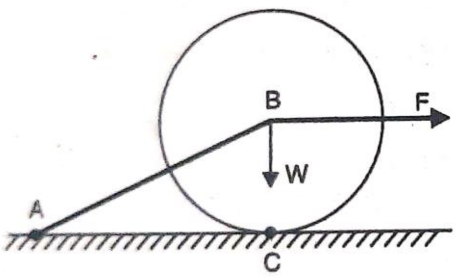


**Fig. 3.42**

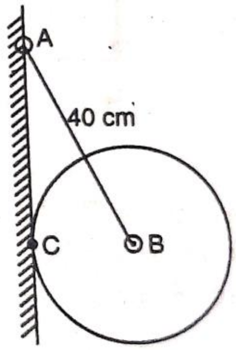


**Fig. 3.43**

**15.** A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB = 40 cm and rests against a smooth vertical wall at C as shown in Fig. 3.43. Determine the tension in the tie rod and reaction $R_C$ at point C.

[**Ans.** 1154.7 N and 577.3 N]

**16.** In problem 6 if radius of ball = 5 cm, length of string AB = 10 cm, weight of ball W= 40 N and the horizontal force F= 30 N, then find the tension the string and vertical reaction $R_C$ at point C.

[**Ans.** 34.64 N and 57.32 N]

**17.** A smooth circular cylinder of weight 1000 N and radius 10 cm rests in a right-angled groove whose sides are inclined at an angle of 30° and 60° to the horizontal as shown in Fig. 3.44. Determine the reaction $R_A$ and $R_C$ at the points of contact,

[**Ans.** RA = 500 N, Rc = 866.6 N]

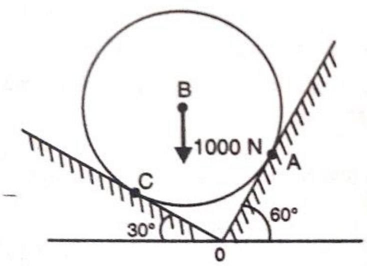


**Fig. 3.44**

**18.** If in the above problem, the sides of the groove makes an angle of 45° with the horizontal, then find the reactions $R_A$ and $R_C$.

[**Ans.** $R_A = R_c = 707$ N]

# 4 FRICTION

**LEARNING OBJECTIVES**

- Concept of Friction
- Limitation Friction and Co-efficient of Friction
- Types of Friction
- Laws of Solid Friction
- Numerical Problems on Sliding Friction
- Angle of Repose
- Sliding Friction on a Rough Inclined Plane

## 4.1 CONCEPT OF FRICTION

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called the force of friction and is always acting in the direction opposite to the direction of motion. The property of the bodies by virtue of which a force is exerted by a stationary body on the moving body to resist the motion of the moving body is called friction. Friction acts parallel to the surface of contact and depends upon the nature of surface of contact.

## 4.2 LIMITING FRICTION AND CO-EFFICIENT OF FRICTION

For defining the terms like co-efficient of friction ($\mu$) limiting friction and angle of friction ($\phi$), consider a solid body placed on a horizontal plane surface as shown in Fig. 4.1.

Let W = Weight of body acting through C.G. downward,
R = Normal reaction of body Acting through C.G. upward,
P = Force acting on the body

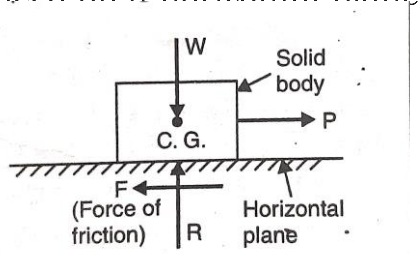


**Fig. 4.1** Solid body on horizontal surface.

Through C.G. and parallel to
the horizontal surface.

If $P$ is small, the body will not move as the force of friction actin on the body in the direction opposite to $P$ will be more than $P$. But if the magnitude of $P$ goes on increasing a stage comes, when the solid body is on the point of motion. At this stage, the force of friction acting on the body is called **limiting force of friction**. The limiting force of friction is denoted by $F$.

Resolving the forces on the body vertically and horizontally, we get

$$R = W$$
$$F = P$$

If the magnitude of $P$ is further increased the body will start moving. The force of friction, acting on the body when the body is moving, is called kinetic friction.

**4.2.1. Co-efficient of Friction (μ)** It is defined as the ratio of the limiting force of friction ($F$) to the normal reaction ($R$) between two bodies. It is denoted by the symbol μ. Thus

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}$$

$$\therefore \qquad F = \mu R \qquad \ldots(4.1)$$

**4.2.2. Angle of Friction(ϕ).**

It is defined as the angle made by the resultant of the normal reaction ($R$) and the limiting force of friction ($F$) with the normal reaction ($R$). It is denoted by ϕ. Fig. 4.2 shows a solid body resting on a rough horizontal plane.

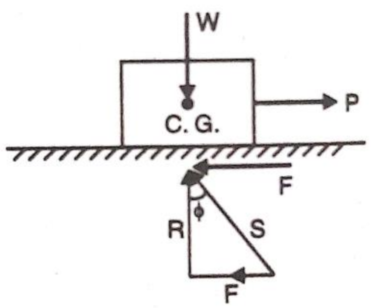


**Fig. 4.2**

Let $S$ = Resultant of the normal reaction ($R$) and limiting force of friction ($F$)

Then angle of friction = ϕ.

= Angle between $S$ and $R$

From Fig. 4.2, we have

$$\tan \phi = \frac{F}{R} = \frac{\mu R}{R} \qquad [\because F = \mu R \text{ from } (4.1)]$$

$$= \mu = \text{Co-efficient of friction} \qquad \ldots(4.2)$$

Thus the tangent of the angle of friction is equal to the co-efficient of friction.

A block of weight $W$ is placed on a rough horizontal plane surface shown in Fig. 4.3 and a force P is applied at an angle θ with the horizontal such that the block just tends to move.

Let $R$ = Normal reaction
$\mu$ = Co-efficient of friction
$F$= Force of friction
$= \mu R$

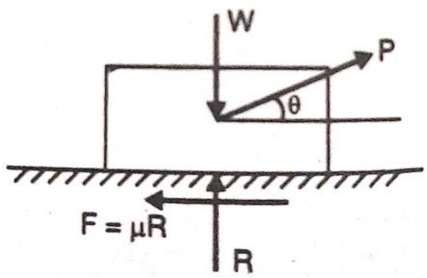


**Fig. 4.3**

In this case the normal reaction $R$ will not be equal to weight of the body. The normal reaction is obtained by resolving the forces on the block horizontally and vertically. The force $P$ in resolved in two components i.e., $P \cos \theta$ in the horizontal direction and $P \sin \theta$ in the vertical direction.

Resolving forces on the block horizontally, we get

$$F = P \cos \theta$$

or $$\mu R = P \cos \theta \qquad \ldots\text{(i)} \quad (\because F = \mu R)$$

Resolving forces on the block horizontally, we get

$$R + P \sin \theta = W$$

$$\therefore \qquad R = W - P \sin \theta$$

From equation (ii), it is clear that normal reaction is not equal to the weight of the block.

If the equation (ii), the value of $W$, $P$ and $\theta$ are known, the value of normal reaction (R) can be obtained. This value of R can be substituted in equation (i) to deter mine the value of co-efficient of friction $\mu$.

**Note.** (i) The force of friction is always equal to $\mu$R (i.e., $F = \mu R$).

(ii) The normal reaction ($R$) is not equal to the weight of the body always,

## 4.3 types of friction

The friction is divided into following two types depending upon the nature of the two surfaces in contact:

1. Static friction, and
2. Dynamic friction.

If the two surfaces, which are in contact, are at rest, the force experienced by one surface is called static friction. But if one surface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction. If between the two surfaces, no lubrication (oil or grease) is used, the friction, that exists between two surfaces is called 'Solid Friction' or 'Dry Friction'.

## 4.1 CONCEPT OF FRICTION

The friction, that exists between two surfaces, which are not lubricated, is known as solid friction. The two surfaces may be at rest or one of the surface is moving and other surface is at rest. The following are the laws of solid friction:

1. The force of friction acts in the opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. When the surface is on the point of motion, the force of friction is maximum and this maximum frictional force is called the limiting friction force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
6. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.
7. The force of friction is independent of the velocity of sliding.

The above laws of solid friction are also called laws of static and dynamic friction.

## 4.5 NUMERICAL PROBLEMS ON SLIDING FRICTION

**Problem 4.1.** A body of weight 100 Newtons is placed on a rough horizontal plane. Determine the co-efficient of friction if a horizontal force of 60 Newtons just causes the body to slide over the horizontal plane.

**Sol.** Given:

Weight of body, $W = 100$ N

Horizontal force applied, $P = 60$ N

∴ Limiting force of friction,

$$F = P = 60 \text{ N}$$

Let $\mu$ = Co-efficient of friction.

The normal reaction of the body is given as

$$R = W = 100 \text{ N}$$

Using equation (4.1),

$$F = \mu R$$

or $\mu = \frac{F}{R} = \frac{60}{100} =$ **0.6. Ans.**

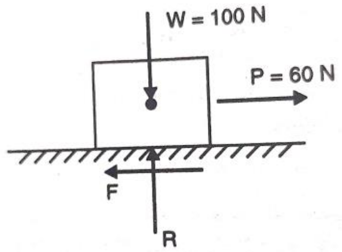


**Fig. 4.4**

**Problem 4.2.** A body of weight 200 N is placed on a rough horizontal plane. If the co-efficient of friction between the body and the horizontal plane is 0.3, determine the horizontal force required to just slide the body on the plane.

**Sol.** Given:
Weight of body, $W = 200$ N
Co-efficient of friction, $\mu = 0.3$
Normal reaction, $R = W = 200$ N

Let $F$ = Horizontal force which causes the body to just slide over the plane.

Using equation (4.1),

$$F = \mu R = 0.3 \times 200 = \mathbf{60\ N.\ Ans.}$$

**Problem 4.3.** The force required to pull a body of weight 50 N on a rough horizontal plane is 15 N. Determine the co-efficient of friction if the force is applied at an angle of 15° with the horizontal.

**Sol.** Given:
Weight of the body, $W = 50$ N
Force applied, $p = 15$ N

Angle made by the force P, with horizontal,
$\theta = 15°$

Let the co-efficient of friction $= \mu$
Normal reaction $= R$

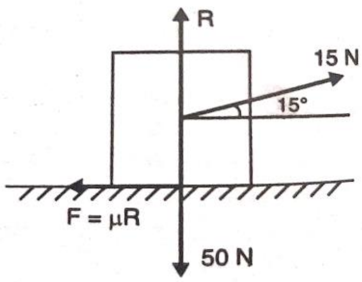


**Fig. 4.5**

When a force equal to 15 N is applied to the body at an angle 15° to the horizontal, the body is on the point of motion in the forward direction. Hence a force of friction equal to $\mu R$ will be acting in the backward direction. The body is in equilibrium under the action of the forces shown in Fig. 4. 5.

Resolving the forces along the plane, $\mu R = 15 \cos 15°$ ...(i)

Resolving the forces normal to the plane

$$R + 15 \sin 15° = 50$$

$$\therefore \quad R = 50 - 15 \sin 15° = 50 - 15 \times 0.2588 = 46.12 \text{ N}$$

Substituting the value of R in equation (i), we get

$$\mu \times 46.12 = 15 \cos 15°$$

$$\therefore \quad \mu = \frac{15 \cos 15°}{46.12} = \frac{15 \times 0.9659}{46.12} = \mathbf{0.314.\ Ans.}$$

**Problm 4.4.** A body of weight 70 N in placed on a rough horizontal plane. To just move the body on the horizontal plane, a push of 20 N inclined at 20° to the horizontal plane is required. Find the co-efficient of friction.

**Sol.** Given:
Weight of body, $W = 70$ N

Force applied $P = 20$ N
Inclined of $P$, $\theta = 20°$

Let $\mu$ = Co-efficient of friction
$R$ = Normal reaction
$F$ = Force of friction = $\mu R$.

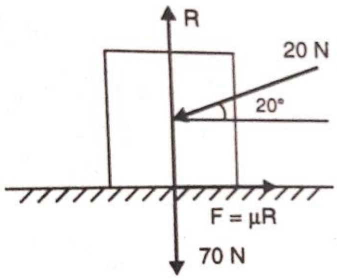


**Fig. 4.6**

When a push of 20 N at an angle 20° to the horizontal is applied to the body, the body is just to move towards left. Hence a force of friction $F = \mu R$, will be acting towards right as shown in Fig. 4.6.

Resolving forces along the plane, $\mu R = 20 \cos 20°$ ...(i)
Resolving forces normal to the plane, $R = 70 + 20 \sin 20°$
$= 70 + 20 \times 0.342 = 70 + 6.84$
$= 76.84$

Substituting the value of $R$ in equation (i),

$$\mu \times 76.84 = 20 \cos 20°$$
$$\mu = \frac{20 \cos 20°}{76.84} = \frac{20 \times 0.9397}{76.84} = \mathbf{0.244.\ Ans.}$$

**Problem 4.5.** A block of weight $W$ is placed on a rough horizontal plane surface as shown in Fig. 4.7 and a force $P$ is applied at an angle $\theta$ with the horizontal such that the block just tends to move. Prove that the force $P$ will be the least if the angle $\theta$ is equal to the angle of friction $\phi$.

Sol. Given:
Weight of block = $W$
Force applied = $P$
Inclination of force = $\theta$

Let $R$ = Normal reaction
$\mu$ = Co-efficient of friction
$F$ = Force of friction = $\mu R$.

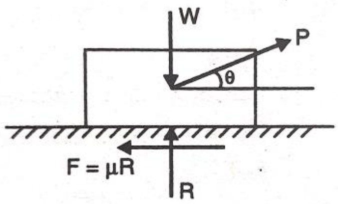


**Fig. 4.7**

The forces acting on the block are shown in Fig. 4.7.

Resolving forces vertically, we get

$$R + P \sin \theta = W$$
or $$R = W - P \sin \theta \qquad \text{...(i)}$$

Resolving forces horizontally, we get

$$P \cos \theta = F = \mu r \qquad (\because F = \mu R) \qquad \text{...(ii)}$$

Substituting the value of R from equation (i), the equation (ii) becomes as

$$P \cos \theta = \mu [W - P \sin \theta] \qquad \text{...(iii)}$$

But from equation (4.2), we know

$$\mu = \tan \phi$$

where $\phi$ = angle of friction.

Substituting the value of $\mu$ in equation (iii), we get

$$P \cos\theta = \tan\phi\,(W - P\sin\theta)$$

$$= \frac{\sin\phi}{\cos\phi}\,(W - P\sin\theta)$$

or $$P\cos\theta\cos\phi = W\sin\phi - P\sin\theta\sin\phi$$

or $P\cos\theta\cos\phi + P\sin\theta\sin\phi = W\sin\phi$

or $P(\cos\theta\cos\phi + \sin\theta\sin\phi) = W\sin\phi$

or $$P\cos(\theta - \phi) = W\sin\phi \quad [\because \cos\theta\cos\phi + \sin\theta\sin\phi = \cos(\theta - \phi)]$$

$$\therefore \quad P = \frac{W\sin\phi}{\cos(\theta - \phi)}$$

The force $P$ will be least, if the denominator i.e., $\cos(\theta - \phi)$ is maximum. But $\cos(\theta - \phi)$ will be maximum, if

$$\cos(\theta - \phi) = 1$$

or $$\theta - \phi = 0$$

or $$\theta = \phi$$

$$\therefore \quad P_{least} = W\sin\phi \text{ or } W\sin\theta$$

Hence the force $P$ will be least if the angle of inclination of $P$ with the horizontal is equal to the angle of friction $\phi$.

**Problem 4.6.** A man wishing to slide a stone block of weight 1000 N over a horizontal concrete floor, ties a rope to the block and pulls it in a direction inclined upward at an angle of 20° to the horizontal. Calculate the minimum pull necessary to slide the block if the co-efficient of friction $\mu$ = 0.6. Calculate also the pull required if the inclination of the rope with the horizontal is equal to the angle of friction and prove that this is the least force required to slide the block.

**Sol.** Given:

Weight, $W$ = 1000 N

Angle with horizontal, $\theta$ = 20°

Co-efficient of friction, $\mu$ = 0.6

Let $P$ = Force applied

$R$ = Normal reaction

$F$ = Force of friction = $\mu R$

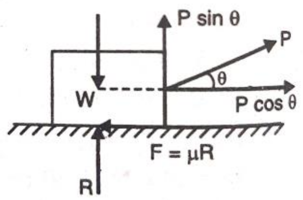


**Fig. 4.8**

The forces acting on the block are shown in Fig. 4.8.

Resolving forces horizontally,

$$P\cos\theta = \mu R$$

or $$P \cos 20° = 0.6 \times R \quad \text{...(i)}$$

Resolving forces vertically, R + P sin θ = $W$

or $R + P \sin 20° = 1000$

or $$R = 1000 - P \sin 20° \quad \text{...(ii)}$$

Substituting the value of $R$ in equation (i), we get

$$P \cos 20° = 0.6\,(1000 - P \sin 20°)$$
$$= 600 - 0.6\, P \sin 20°$$

or $$P \cos 20° + 0.6\, P \sin 20° = 600$$

or $$P\,(\cos 20° + 0.6 \times \sin 20°) = 600 \quad \text{...(iii)}$$

$$\therefore \quad P = \frac{600}{(\cos 20° + 0.6 \sin 20°)} = \frac{600}{(0.9397 + 0.6 \times 0.342)}$$

$$= \frac{600}{1.1449} = \mathbf{524\ N.\ Ans.}$$

Pull required if the inclination of the rope with the horizontal is equal to angle of friction.

Let ϕ = angle of friction

= The angle made by rope with horizontal (given) = 20°

If in equation (iii), the angle 20° is replaced by angle ϕ, then we get the force required to pull the body as,

$$P\,(\cos \phi + 0.6 \sin \phi) = 600$$

$$\therefore \quad P = \frac{600}{(\cos \phi + 0.6 \sin \phi)}$$

The force P will be minimum, if (cos ϕ + 0.6 sin ϕ) is maximum.

But (cos ϕ + 0.6 sin ϕ) will be maximum if

$$\frac{d}{d\phi}(\cos \phi + 0.6 \sin \phi) = 0 \quad \text{or} \quad -\sin \phi + 0.6 \cos \phi = 0$$

or $$0.6 \cos \phi = \sin \phi \quad \text{or} \quad 0.6 = \frac{\sin \phi}{\cos \phi} = \tan \phi$$

But 0.6 = μ. Hence force P will be minimum if tan ϕ = μ = 0.6 (Proved)

Now $$\tan \phi = 0.6$$

$$\therefore \quad \phi = \tan^{-1} 0.6 = 30.96°$$

Substituting this value of ϕ in equation (iv), we get

$$P = \frac{600}{(\cos 30.96° + 0.6 \sin 30.96°)} = \frac{600}{(0.8575 + 0.6 \times 0.5144)}$$

$$= \frac{600}{1.1661} = \mathbf{514.5\ N.\ Ans.}$$

## 4.6 ANGLE OF REPOSE

The angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Consider a body of weight W, resting on a rough inclined plane as shown in Fig. 4.9.

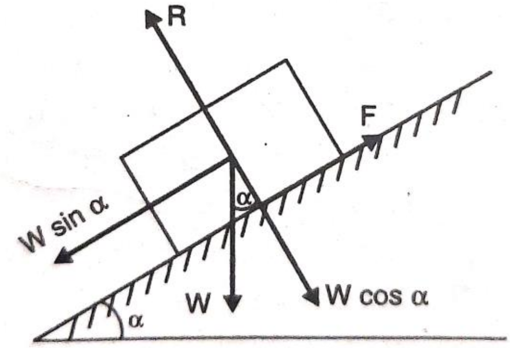


**Fig. 4.9**

Let R = Normal reaction acting at right angle to the inclined plane.

$\alpha$ = Inclination of the plane with the horizontal

F = Frictional force acting upward along the plane.

Let the angle of inclination ($\alpha$) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called angle of repose.

Resolving the forces along the plane, we get

$$W \sin \alpha = F \quad \ldots(i)$$

Resolving the forces normal to the plane, we get

$$W \cos \alpha = R \quad \ldots(ii)$$

Dividing equation (i) by equation (ii),

$$\frac{W \sin \alpha}{W \cos \alpha} = \frac{F}{R} \text{ or } \tan \alpha = \frac{F}{R} \quad \ldots(iii)$$

But from equation (4.2), we know

$$\tan \phi = \frac{F}{R} \quad \ldots(iv)$$

Where $\phi$ = angle of friction.

Hence from equations (iii) and (iv), we have

$$\tan \alpha = \tan \phi$$

or $$\alpha = \phi$$

or Angle of repose = Angle of friction.

## 4.7 SLIDING FRICTION ON A ROUGH INCLINED PLANE

In art, 4.5 we have studied that if the inclination of the plane, with the horizontal, is less than the angle of friction, the body will remain in equilibrium without any external force. If the body is to be moved upwards or downwards in this condition on friction external force is required. But if the inclination of the plane is more than the angle of friction, the body will not remain in equilibrium. The body will move downward and an upward external force will be required to keep the body in equilibrium.

Such problems are solved by resolving the forces along the plane and perpendicular to the planes. The force of friction (F), which is always equal to $\mu R$ is a acting opposite to the direction of motion of the body.

**Problem 4.7.** Prove that the angle of friction ($\phi$) is equal to the angle made by an inclined plane with the horizontal when a solid body, placed on the inclined plane, is about to slide down.

**Sol**. A solid body of weight, W is placed on an inclined plane AC as shown in Fig.4.10.

Let $\alpha$ = Angle of the inclined plane AC with horizontal plane AB, such that body just starts moving downward.

The body is in equilibrium under the action of following forces:

1. Weight of the body (W) acting vertically downwards.
2. Normal reaction (R), acting perpendicular to the inclined plane, AC.
3. The force of friction, F = $\mu R$, acting up the plane as the body is about to slide down the plane.

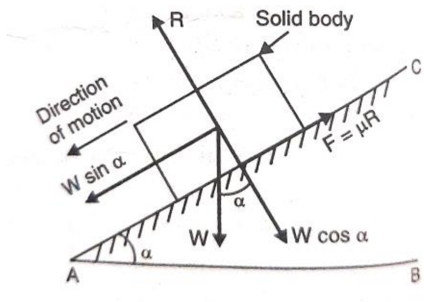


**Fig. 4.10**

The weight, W can be resolved in two component one along the plane and other perpendicular to the plane. The components are W sin $\alpha$ W cos $\alpha$ respectively.

As the body is in equilibrium, the forces along the perpendicular to the inclined plane are :

$$W \sin \alpha = F = \mu R$$

$$W \cos \alpha = R$$

Dividing $$\frac{W \sin \alpha}{W \cos \alpha} = \frac{\mu R}{R} = \mu$$

or $$\tan \alpha = \mu$$

But from equation (4.2), we have

$$\tan \phi = \mu$$

Where $\phi$ = angle of friction

$\therefore$ $$\tan \alpha = \text{Tan } \phi = \mu$$

or $$\alpha = \phi.$$

The above relation shows that the angle of friction is equal to angle of the inclined plane when a solid body, placed on the inclined plane is about to slide down.

**Problem 4.8.** a body of weight 500 N is pulled up an inclined plane, by a force of 350 N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the co-efficient of friction.

**Sol.** Given :

Weight of body, $W = 500$ N

Force applied, $P = 350$ N

Inclination, $\alpha = 30°$

Let $\mu$ = Co-efficient of Q friction

$R$ = Normal reaction

$F$ = Force of friction = $\mu R$.

The body is in equilibrium under the action of the forces shown in Fig. 4.11.

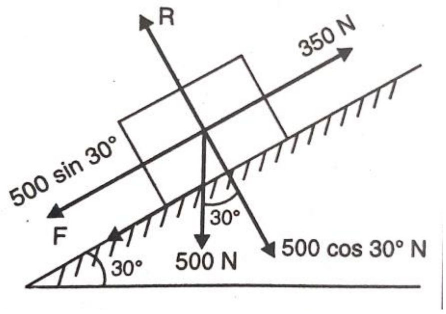


**Fig. 4.11** Body moving up

Resolving forces along the plane,

$$500 \sin 30° + F = 350$$

or $$500 \sin 30° + \mu R = 350$$

Resolving forces normal to the plane,

$$R = 500 \cos 30° = 500 \times .866 = 433 \text{ N}$$

Substituting the value of R in equation (i), we get

$$500 \sin 30° + \mu \times 433 = 350$$

or $$500 \times 0.5 + 433\,\mu = 350$$

or $$433\,\mu = 350 - 500 \times 0.5 = 350 - 250 = 100$$

$\therefore$ $$\mu = \frac{100}{433} = 0.23. \text{ Ans.}$$

**Problem 4.9.** A body of weight 450 N is pulled up along an inclined plane having inclination 30° to the horizontal at a steady speed. Find the force required if the co-efficient of friction between the body and the plane is 0.25 and force is applied parallel to the inclined plane. If the distance travelled by the body is 10 m along the plane, find the work done on the body.

**Sol.** Given :

Weight of body, $W = 450$ N

Inclination of plane, $\alpha = 30°$

Co-efficient of friction, $\mu = 10.25$

Distance travelled by body $= 10$ m

Let the force required $= P$.

The body is equilibrium under the action of forces shown in Fig. 4.12.

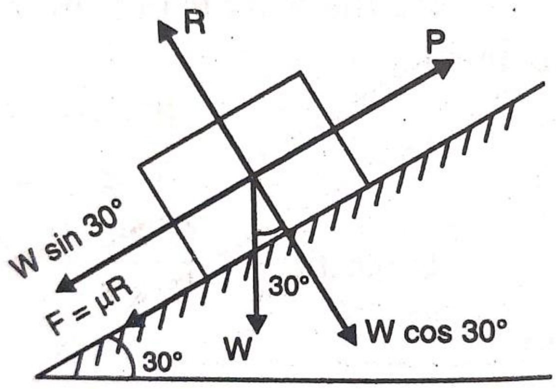


**Fig. 4.12** Body moving up

Resolving forces along the plane,

$$P = W \sin 30° + \mu R = 450 \times 0.5 + 0.25 \times R$$

or $$P = 225 + 0.25 R \quad \ldots(i)$$

Resolving forces normal to the plane,

$$R = W \cos 30° = 450 \times 0.866 = 389.7 \text{ N}$$

Substituting the value of R in equation (i),

$$P = 225 + 0.25 \times 389.7 = 322.425 \text{ N. Ans.}$$

Work done on the body = Force × Distance travelled in the direction of force

$$= 322.525 \times 10 \text{ Nm} = 3224.25 \text{ Nm}$$

$$= 3224.25 \text{ J (where J = Joules = Nm). Ans.}$$

**Problems .4.10.** (a) Define co-efficient of friction and limiting friction. (b) Block A weighing 15 N is a rectangular prism resting on a rough inclined plane as shown in Fig. 4.13. the block is tied up by a horizontal string which has a tension of 5 N. Find :

i. The frictional force on the block,
ii. The normal reaction of the inclined plane, and
iii. The co-efficient of friction between the surface of contact.

**Sol.** (a) For definition of co-efficient of friction and limiting friction, please refer to Art. 4.2 and Art 4.2.1.

(b) Given :

Weight of block, W = 15 N

Tension in string, T = 5 N

Inclination of plane, $\alpha = 45°$

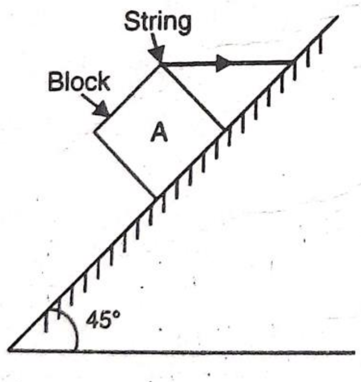


**Fig. 4.13**

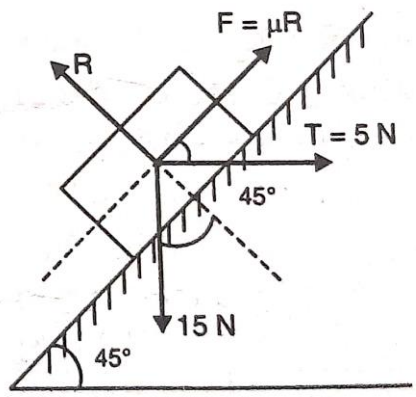


**Fig. 4.14** Forces acting on the block

Let F = Frictional force,

R = Normal reaction, and

μ = Co-efficient of friction.

Since there is tension in string which means if string is removed, the block will slide down the plane. Hence force of friction will be acting in the upward direction.

The block A is in equilibrium under the action of the forces shown in Fig.4.14. The forces are :

1. The weight of block, W = 15 N
2. Horizontal tension in the string, T = 5 N
3. Normal reaction, R
4. Force of friction, F = μR1 acting upward. Resolving forces along the inclined plane,

$$15 \sin 45° = F + 45°$$

∴ $F = 15 \sin 45° - 5 \cos 45° = 15 \times .707$

$= 10 \times .707 = 7.07$ N

Resolving forces normal to inclined plane,

$R = 15 \cos 45° + T \text{ Cos } 45° = 15 \cos 45° + 5 \cos 45°$

$= 15 \times .707 + 5 \times .707 = 14.14$ N

Using equation (4.1), we get

$F = \mu R$

∴ $\mu = \frac{F}{R} = \frac{7.07}{14.14} = 0.5.$

i. Frictional force on the block, F = 7.07 N. Ans.
ii. Normal reaction of the inclined plane, R = 14.14 N. Ans.
iii. Co-efficient of friction, μ = 0.5. Ans.

**Problem 4.11.** Find the force required to move a load of 30 N up a rough inclined plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same body is kept on a perfectly smooth plane inclined at that angle, a force of 6 N applied at an inclination of 30° to the plane keeps the same in equilibrium. Assume co-efficient of friction between the rough plane and the load is equal to 0.3.

**Sol.** Given :

Load, W = 30 N

Co-efficient of friction between the rough plane and load,

μ = 0.3

Let $\alpha$ = Inclination of the plane with horizontal

$P_1$ = Force required to move the load up a rough inclined plane, when the force is applied parallel to the plane.

The force is applied when same body is kept On a smooth inclined plane, $P_2$ = 6 N.

Inclination of the force with the inclined plane, θ = 30°.

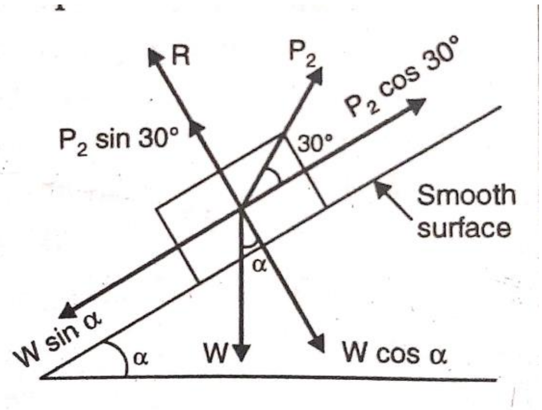


**Fig. 4.15**

**1st case.** Consider the body of weight 30 N placed on a smooth inclined plane as shown in Fig. 4.15.

The force acting on the body are :

i. The weight (W = 30 N) vertically downward.
ii. The force $P_2$(= 6N) at an angle of 30° with the inclined plane.
iii. The normal reaction R.

Resolving forces normal to the inclined plane.

$$R + P_2 \sin 30° = W \cos \alpha$$

or $$R + 6 \times \frac{1}{2} = 30 \cos \alpha \quad \text{...(i)} \; (\therefore P2 = 6\text{ N}, W = 30\text{ N})$$

Resolving forces along the inclined plane,

$$P_2 \cos 30° = W \sin \alpha$$

or $$6 \times \frac{\sqrt{3}}{2} = 30 \sin \alpha$$

∴ $$\sin \alpha = 6 \times \frac{\sqrt{3}}{2} \times \frac{1}{30} = \frac{\sqrt{3}}{10} = 0.1782$$

$\alpha$ = 9.974 . **Ans.**

**2nd case.** The body of weight 30 N is placed on a rough inclined plane having inclination α(= 9.974°) with the horizontal as shown in fig. 4. 16.

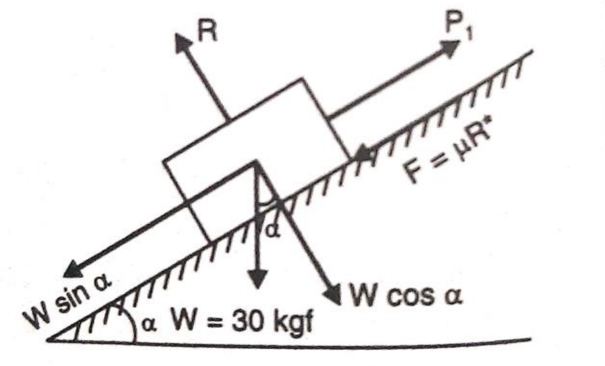


**Fig. 4.16**

The forces acting on the body are:

i. The weight W(= 30 N) vertically downward.
ii. The force $P_1$ , parallel to the plane.
iii. The normal reaction R*.
iv. Force of friction F = μR*.

Resolving forces along the inclined plane,

$$W \sin \alpha + F = P_1$$

or $\quad 30 \sin 9.974° + \mu R^* = P_1 \quad$ ($\therefore$ F = μR* and α = 9.974°) ...(ii)

Resolving forces normal to the inclined plane

$$R^* = W \cos \alpha$$

$$= 30 \times \cos 9.974° \quad (\therefore \alpha = 9.974°)$$

$$= 30 \times 9.9848 = 29.544.$$

Substituting the value of R* in equation (ii), we get

$$30 \sin 9.974 + \mu \times 29.544 = P1$$

or $\quad 30 \times .1732 + 0.3 \times 29.544 = P1$

or $\quad 5.196 + 8.8632 = P1$

or $\quad P1 = 14.059$ N. Ans.

**Problem 4.12.** Two blocks A and B are connected by a horizontal rod and are supported on two rough planes as shown in Fig. 4.17. if the weight of block B is 1500 N and co-efficient of friction of block A and B are 0.25 and 0.35 respectively. Find the smallest weight of block A for which equilibrium can exist.

**Sol.** Given : Weight of block B,

$$W_\beta = 1500\ N$$

Co-efficient of friction for block A,

$$\mu_A = 0.25$$

Co-efficient of friction for block B,

$$\mu\beta_\beta = 0.35$$

Let the smallest weight of block A for equilibrium

$$= W/_A$$

If the weight of block A is less than the value required for equilibrium, the block B will be slide downwards. But the block A and B are connected by a horizontal rod of fixed length. Now when blocks B starts moving in the downward direction, the block A starts moving towards left. Hence a force of friction F A equal to $\mu_A$ $R_A$ will be acting on block A towards right as shown in Fig.4.17.

On block B, the force of friction $F_\beta$ equal to $\mu_\beta$ $R_\beta$ will be acting in the upward direction.

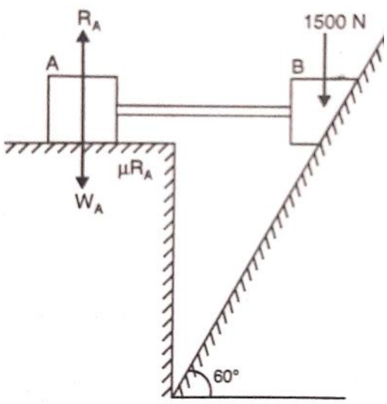


**Fig. 4.17**

**For block A**

Resolving force normal to plane, $R_A = W_A$.

Force of friction, $F_A = \mu_A R_A = \mu\ 0.25 \times W_A = 0.25\ W_A$

This force will be transmitted to block B through rod AB.

**For block B**

The block B will be equilibrium under the action of the forces, shown in fig. 4.18. the forces are :

i. The weight of block B = 1500 N acting vertically downwards.
ii. The normal reaction $R_\beta$ of the plane.
iii. The horizontal force = 0.25 WA' transmitted to block B through rod AB.
iv. Force of friction $F_\beta = \mu_\beta R_\beta = 0.35 R_\beta$ acting up the inclined plane.

In this case, the forces are resolved horizontally and vertically instead of along the inclined plane and normal to the plane. For an equilibrium state, the forces acting in any direction must be zero.

Resolving forces horizontally,

$$0.25 W_A + F_\beta \cos 60° = R_\beta \cos 30°$$

$$0.25 W_A + 0.35 R\text{ß} \cos 60° = R_\beta \cos 30° \qquad (\therefore F_\beta = 0.35 R_\beta)$$

$$0.25 W_A + 0.35 \times 0.5 R_\beta = R_\beta \times .866$$

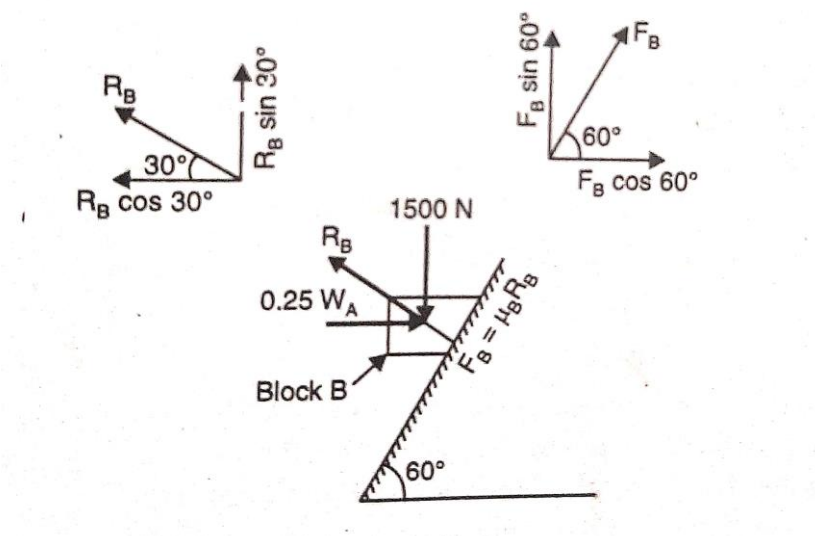


**Fig. 4.18**

$$0.25 W_A + 0.175 R_\beta = 0.866 R_\beta$$

or $$0.25 W_A = 0.866 R_\beta - 0.175 R_\beta = 0.691 R_\beta \qquad \ldots(i)$$

Resolving forces vertically,

$$R_\beta \sin 30° + F_\beta \sin 60° = 1500$$

$$R_\beta \times 0.5 + 0.35 R_\beta \times 0.866 R\text{ß} = 1500 \qquad (\therefore F_\beta = 0.35 R_\beta)$$

$$0.5 R_\beta + 0.303 R_\beta = 1500 = 0.803 R_\beta = 1500$$

$\therefore \qquad R_\beta = \frac{1500}{0.803} = 1868$ N.

Substituting the value of Rß in equation (i), we get

$$0.25\ W_A = 0.691 \times 1868$$

$\therefore \qquad W_A = \frac{0.691 \times 1868}{0.25} = 5163$ N. Ans.

**Problem 4.13.** Refering to the Fig. 4.19 given below, determine the least value of the force P to cause motion to impend rightwards. Assume the co-efficient of friction under the blocks to be 0.2 and pulley to be frictionless.

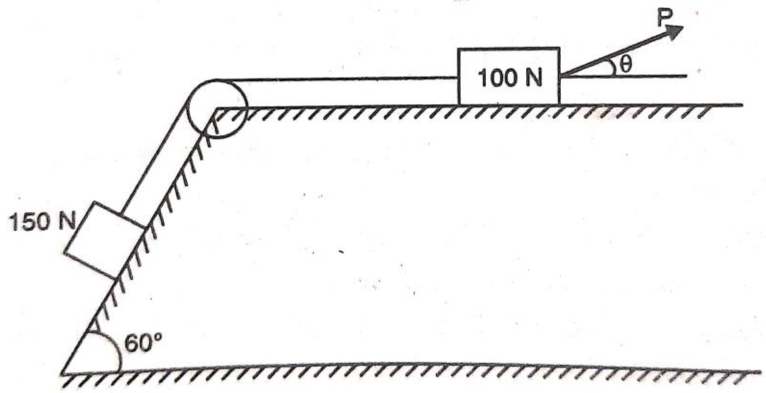


**Fig. 4.19**

**Sol.** Given :

Co-efficient of friction under both blocks, $\mu = 0.2$

Pulley is frictionless. Motion of block of weight 100 N is towards right. Find least value of P.

**1st case**

Consider the equilibrium of block of weight 150 N

As the block of weight 100 N tends to move rightwards, the block of weight 150 N will tend to be move upwards. Hence force of friction will act downwards as shown in Fig 4.20.

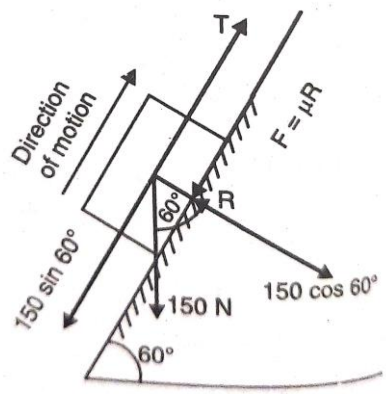


**Fig. 4.20**

Let $T$ = Tension in the string

$R$ = Normal reaction

$F$ = Force of friction = $\mu R$

$= 0.2R$

The weight 150 N is acting vertically downwards. The body is in equilibrium under the action of the forces, shown in fig. 4.20. Resolving forces along the plane,

$$T = 150 \sin 60° + \mu R$$

Resolving forces normal to the inclined plane,

$$R = 150 \cos 60° = 150 \times \frac{1}{2} = 75 \text{ N} \quad \text{...(ii)}$$

Substituting the value of R in equation (i),

$$T = 150 \sin 60° + 0.2 \times 75 \quad (\therefore \ \mu = 0.2)$$

$$= 144.9 \text{ N}$$

**2nd case**

Now consider the equilibrium of block of weight 100 N

The block of 100 N tends to move rightwards, hence force of friction will be acting towards left as show Fig. 4.21. Also the pulley is frictionless hence the tension in the string which is attached to the block of weight 100 N will be 144.9 N. The body is in equilibrium under the action of the forces, shown in fig. 4.21.

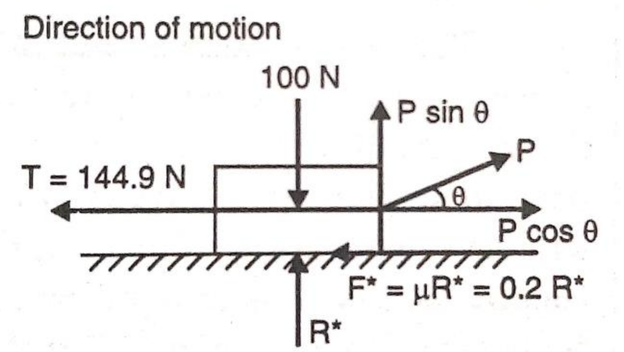


**Fig. 4.21**

Resolving forces along the plane,(i.e., horizontally),

$$P \cos \theta = T + 0.2 R^*$$

$$= 144.9 + 0.2 R^* \quad \text{...(iii)}$$

Resolving forces normal to the plane, (i.e., vertically),

$$R^* + P \sin \theta = 100$$

or $$R^* = 100 - P \sin \theta \quad \text{...(iv)}$$

Substituting the value of R* in equation (iii),

$$P \cos \theta = 144.9 + 0.2 \times (100 - P \sin \theta) = 144.9 + 20 - 0.2 P \sin \theta$$

or $$P \cos \theta + 0.2 \sin \theta = 164.9$$

or $$P (\cos \theta + 0.2 \sin \theta) = 164.9$$

or $$P = \frac{164.9}{(\cos \theta + 0.2 \sin \theta)} \quad \text{...(v)}$$

Thus force P will be minimum, if $(\cos \theta + 0.2 \sin \theta)$ is maximum.

But $(\cos \theta + 0.2 \sin \theta)$ will be maximum if

$$\frac{d}{d\theta} (\cos \theta + 0.2 \sin \theta) = 0$$

or $$- \sin \theta + 0.2 \cos \theta = 0$$

or $$0.2 \cos \theta = \sin \theta$$

or $$0.2 = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

or $$\theta = \tan^{-1} 0.2 = 11.309°$$

Substituting the value of θ in equation (v), the least value of P will be obtained.

$$\therefore \quad P_{(least)} = \frac{164.9}{(\cos 11.309° + 0.2 \times \sin 11.309)} = \mathbf{161.88\ N.\ Ans.}$$

**Problem 4.14.** What should be the value of the angle θ in Fig. 4.22 so that the motion of the 90 N block impends down the plane. The co-efficient of friction μ for all the surface is 1/3.

**Sol.** Given :

Co-efficient of friction for all surfaces, μ = 1/3

Motion of weight 90 N impends down the plane. Find the value of θ.

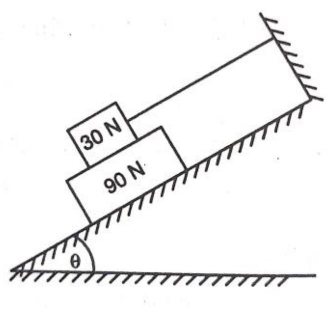


**Fig. 4.22**

*First consider the equilibrium of weight 30 N.*

As the weight 90 N tends to move downwards, there will be a rubbing action between the surface of weight 90 N and 30 N. Hence a force of friction will be acting between these two surfaces.

The weight 30 N is tied to a string, the other end of the string is fixed to the plane. When the weight 90 N tends to move downwards, the weight 30 N with respect to 90 N will move upwards. Hence the force of friction on the lower surface of the weight 30 N will act downward as shown in Fig. 4.22 (a). The weight 30 N will be in equilibrium under the action of the forces, shown in fig. 4.22(a) in which

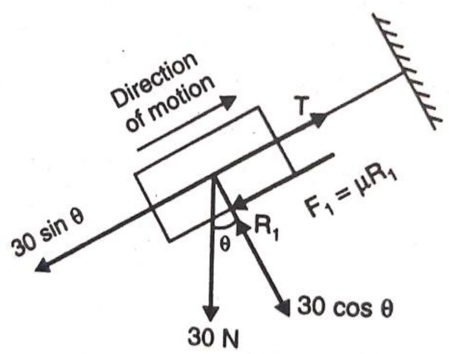


**Fig. 4.22(a)**

$T$ = Tension in the string

$R_1$ = Normal reaction on the lower surface of weight 30 N

$F_1$ = Force of friction = $\mu R_1$

Resolving forces along the plane,

$$T = 30 \sin\theta + \mu R_1$$

$$= 30 \sin\theta + \frac{1}{3} R_1 \quad \ldots\text{(i)}$$

Resolving forces normal to the plane,

$$R1 = 30 \cos\theta \quad \ldots\text{(ii)}$$

Substituting the value of R1 in equation (i),

$$T = 30 \sin\theta + \frac{1}{3} \times 30 \cos\theta \quad \ldots\text{(iii)}$$

Now consider the equilibrium of weight 90 N

The weight 30 N will be in equilibrium under the action of the forces, shown in fig. 4.22(b).

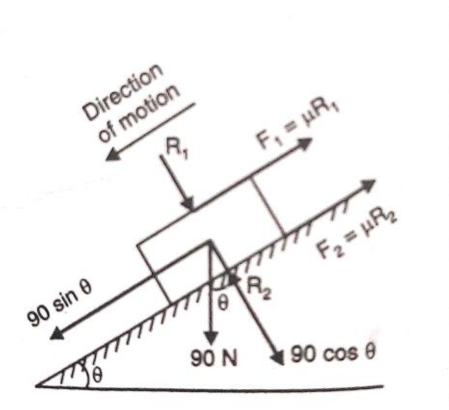


**Fig. 4.22(b)**

Resolving forces along the plane,

$$90 \sin \theta = \mu R_1 + \mu R_2$$

$$= \frac{1}{3} R_1 + \frac{1}{3} R_2$$

$$= \frac{1}{3} \times 30 \cos \theta + \frac{1}{3} R_2 \qquad (\because \; R_1 = 30 \cos \theta)$$

$$= 10 \cos \theta + \frac{1}{3} R_2 \qquad \ldots(\text{iv})$$

Resolving forces normal to the plane,

$$R_2 = R1 \quad 90 \cos \theta$$

$$= 30 \cos \theta + 90 \cos \theta \qquad (\because \; R_1 = 30 \cos \theta)$$

$$= 120 \cos \theta \qquad \ldots(\text{v})$$

Substituting the value of R2 in equation (iv), we get

$$90 \sin \theta = 10 \cos \theta + \frac{1}{3} \times 120 \cos \theta$$

$$= 10 \cos \theta + 40 \cos \theta = 50 \cos \theta$$

$$\therefore \quad \frac{\sin \theta}{\cos \theta} = \frac{50}{90} = 0.5555$$

or $$\tan \theta = 0.5555$$

$$\therefore \quad \theta = \tan^{-1} 0.5555 = \mathbf{29.05^\circ}. \textbf{ Ans.}$$

## STUDENT ACTIVITY

**1.** Explain the difference between co-efficient of friction and angle of friction.

**2.** Define static and kinetic friction and state the laws of solid friction.

# SUMMARY

1. Force of friction always acts in the direction opposite to the direction of motion.
2. The maximum value of frictional force acting on a body, when the body is on the point of motion, is called limiting force of friction. It is denoted by F.
3. The force of friction, acting on a body when the body is moving, is called dynamic friction.
4. The ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies is known as co-efficient of friction. It is denoted by $\mu$. Mathematically, $\mu = \frac{F}{R}$.
5. The angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction is known as angle of friction. It is denoted by $\phi$.
6. The relation between angle of friction ($\phi$) and co-efficient of friction ($\mu$) is expressed as $\tan \phi = \mu$.
7. The force of friction always equal to $\mu R$, where R is normal reaction.
8. If a body is placed on a rough inclined plane and the angle of inclination of the plane is gradually increased, till the body just starts sliding down the plane. The angle of the inclined plane, at which the body just begins to slide down the plane, is called angle of repose.
9. Angle of repose is equal to angle of friction.
10. If the inclination of the plane, with the horizontal is less than angle of friction, the body is placed on the inclined plane will be always in equilibrium without any external force.

# TEST YOURSELF

### (A) Theoretical Question

1. Define the following terms : friction, limiting force of friction, co-efficient of friction and angle of friction.
2. Explain the difference between, co-efficient of friction and angle of friction.
3. (a) State the laws of static and dynamic friction.
   (b) State the laws of solid friction.
4. Prove that the angle of friction is equal to the angle of inclined plane, when a solid body of weight W placed on the inclined plane, is about to slide down.
5. What do you mean by 'angle of repose'? prove that angle of repose is equal to the angle of friction.

**6.** A body of weight W is placed on an inclined plane. The inclination of the plane with the horizontal is less than the angle of friction. The body will

(a) be in equilibrium (b) move downwards

(c) move upwards (d) none of the above [**Ans.** (a)]

**7.** f in the above question, the inclination of the plane with the horizontal is more than the angle of friction, the body will

(a) be in equilibrium (b) move downwards

(c) move upwards (d) none of the above [**Ans**. (b)]

**8.** A body of weight W is placed on a rough inclined plane having inclination α to the horizontal. The force P is applied horizontally to drag the body . if the body is on the point of motion up the plane, prove that P is given by

$$P = W \tan (\alpha - \phi)$$

Where $\phi$ = Angle of friction.

**9.** In the above question, if the body is on the point of motion down the plane, prove that the force P is given by

$$P = W \tan (\alpha - \phi).$$

**10.** Define static and kinetic friction and state the laws of solid friction.

## (B) Numerical Questions

**1.** A body of weight 90 N is placed on a rough horizontal plane. Determine the co-efficient of friction if a horizontal force of 63 N just causes the body to slide over the horizontal plane. [**Ans.** 0.7]

**2.** A body of weight 150 N is placed on a rough horizontal plane. If the co-efficient of the friction between the body and the horizontal plane is 0.4 , determine the horizontal force required to just slide the body on the plane.

[**Ans.** 60 N]

**3.** The force pull required the body of weight 40 N on a rough horizontal plane 15 N. Determine the co-efficient friction of the force is applied at an angle of 20° with the horizontal.

[**Ans.** 0.404]

**4.** A body of weight 60 N is placed on a rough horizontal plane to just move the body on the horizontal plane, a push of 18 N inclined at 20° to the horizontal plane is required. Find the co-efficient of friction.

[**Ans.**0.255]

**5.** Find the least force required to pull a body of weight W placed on a rough horizontal plane, when the force is applied at an angle θ with the horizontal.

[**Ans.** W sin θ]

**6.** A body of weight 450 N is pulled up an inclined plane, by a force of 300 N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the coefficient of friction.

[**Ans.**192]

**7.** A body of weight 400 N is pulled up along an inclined plane having inclination 30° to the horizontal at a steady speed. If the co-efficient of friction between the body and the plane is 0.3 and force is applied parallel to the inclined plane, find the force required. Find also the work done on the body if the distance travelled by the body is 10 m along the plane.

[**Ans.**303.92 N, 3039.2 Nm]

**8.** Block a weighing 20 N is a rectangular prism resting on a rough inclined plane as shown in Fig. 4.20. A block is tied up by a horizontal string which has a tension of ß N, Find : (i) the frictional force on the block, (ii) the normal reaction of the inclined plane, and (iii) the co-efficient of friction between the surface of contact.

[**Ans.** (i) 9.808 N (ii) 18.382 N (iii) 0.598]

**9.** A body of weight 100 N is at rest on a horizontal plane. A horizontal force of 70 N just causes the body to slide. Determine : (i) limiting force of friction and (ii) co-efficient of friction.

[**Ans.** (i) 70 N, (ii)0.7]

**10.** The co-efficient of friction between a body of weight 100 N and the rough horizontal plane on which the body rests is 0.3. Calculate the horizontal force required just to cause the body to slide over the horizontal plane.
If the body is loaded with an additional weight of 40 N, find the least horizontal force which will the cause the body to side.

[**Ans.** (i)30 N, (ii)45 N]

# 5 CENTRE OF GRAVITY

**LEARNING OBJECTIVES**

- Centre of Gravity
- Centroid
- Centroid or Centre of Gravity of Simple Plane Figure
- Centre of Gravity of Plane Figure by the Method of Moments
- Centre if Gravity of Bodies with Portion Removed

## 5.1 CENTRE OF GRAVITY

Centre of gravity of a body is the point through which the whole weight of the body acts. A body is having only one centre of gravity for all positions of body. It is represented by C.G or simply G

## 5.2 CENTROID

The point at which the total area of a plane figure (like rectangle, square, triangle, quadrilateral, circle etc.) is assumed by concentrated, is known as the centroid of that area. The centroid is also represented by C.G or simply G. The centroid and centre of gravity are at the same point.

## 5.3 CENTROID OR CENTRE OF GRAVITY OF SIMPLE PLANE FIGURES

i. The centre of gravity (C.G) of a uniform rod lies at its middle point.
ii. The centre of gravity of a triangle lies at the point where the three medians* of the triangle meet.
iii. The centre of gravity of a rectangle or of a parallelogram is at the point, where is diagonal meet each other. It is also the point of intersection of the lines joining the middle points of the opposite sides.
iv. The centre of gravity of a circle is at its centre.

## 5.4 CENTER OF GRAVITY OF PLANE FIGURES BY THE METHOD OF MOMENTS

Fig 5.1 shows a plane figure of total area A whose centre of gravity is to be determined. Let this area A is composed of a number of small areas $a_1$, $a_2$, $a_3$, $a_4$, ……. etc.

∴ A = $a_1$, $a_2$, $a_3$, $a_4$, + …….

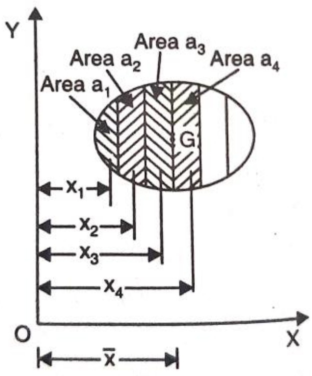


**Fig. 5.1**

Let Let $x_1$ = The distance of the C.G of the area $a_1$ from axis OY

$x_2$ = The distance of the C.G of the area $a_2$ from axis OY

$x_3$ = The distance of the C.G of the area $a_3$ from axis OY

$x_4$ = The distance of the C.G of the area $a_4$ from axis OY and so on.

The moments of all small areas about the axis OY

$= a_1x_1 + a_2x_2 + a_3x_2 + a_4x_4 + \ldots.$ ….(i)

Let G the centre of gravity of the total area A whose distance from the axis OY is $\bar{x}$.

Then moment of total area about OY = A$\bar{x}$ ….(ii)

The moments of all small areas about the axis OY must be equal to the moment of total area about the same axis. Hence equating equations (i) and (ii), we get

$$= a_1x_1 + a_2x_2 + a_3x_2 + a_4x_4 + \ldots. = A\bar{x}$$

or $$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_2 + a_4x_4 + \ldots.}{A}$$ ….(5.1)

where A = $a_1 + a_2 + a_3 + a_4$

If we take the moments of the small areas about the axis OX and also moment of total area about this axis OX, we will get

$$\overline{y} = \frac{a_1y_1 + a_2y_2 + a_3y_2 + a_4y_4 + ....}{A} \qquad ....(5.2)$$

where $\overline{y}$ = The distance of G from axis OX

$y_1$ = The distance of C.G of the area $a_1$ from axis OX

$y_2 y_3\ y_4$ = The distance of C.G of area $a_1$ from axis OX respectively.

**5.4.1. Centre of Gravity of Plane Figures by Integration Method.** The equations (5.1) and (5.2) can be written as

$$\overline{x} = \frac{\sum a_i x_i.}{\sum a_i} \text{ and } \overline{y} = \frac{\sum a_i y_i.}{\sum a_i}$$

where $i$ =1,2,3,4,....

$x_1$ = Distance of C.G of area a1 from axis OY and

$y_1$ = Distance of C.G of area a1from axis OX.

The value of i depends upon the number of small areas. If the small areas are large in number (mathematically speaking infinite in number),then the summations in the above equations can be replace by integration. Let them small areas are represented by $d$A instend of '$a$', then the above equations are written as :

$$\overline{x} = \frac{\int x* \, dA}{\int dA} \qquad ...[5.2(A)]$$

and

$$\overline{y} = \frac{\int y* \, dA}{\int dA} \qquad ...[5.2(B)]$$

where $\int x * \ dA = \sum a_i x_i$

$\int dA = \sum a_i$

Also, $x*$ = Distance of C.G of area $dA$ from OY

$y*$ = Distance of C.G of area $dA$ from OX

**5.4.2.Centre of Gravity of a Line.** The centre of gravity of a line which may be straight or curve, is obtained by diving the given line, into a large number of small lengths as shown in Fig.5.1 (a)

The centre of gravity is obtained by replacing $dA$ by $dL$ in equations [5.2 (A)] and [5.2 (B)].

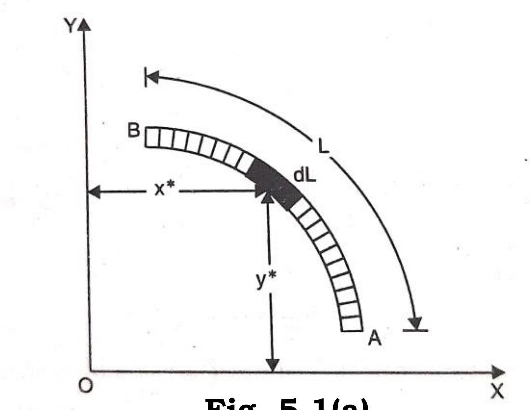


**Fig. 5.1(a)**

Then these equations become $\overline{x} = \frac{\int x * \, dL}{\int dA}$ ...[5.2(C)]

and $\overline{y} = \frac{\int x * \, dL}{\int dL}$ ...[5.2(D)]

Where $x^*$ = Distance of C.G of length of $dL$ from y-axis, and

$y^*$ = Distance of C.G of length of $dL$ from x-axis

If the lines are straight, then the above equations are written as :

$$\overline{x} = \frac{L_1x_1 + L_2x_2 + L_3x_2 + \ldots\ldots\ldots\ldots}{L_1 + L_2 + L_3 + \ldots\ldots\ldots\ldots} \qquad \ldots[5.2(E)]$$

and $$\overline{y} = \frac{L_1y_1 + L_2y_2 + L_3y_2 + \ldots\ldots\ldots\ldots}{L_1 + L_2 + L_3 + \ldots\ldots\ldots\ldots} \qquad \ldots[5.2(F)]$$

### 5.4.3. Important Points For Centre of Gravity

i. The axis, about which moments of areas are taken, is known as axis of reference. In the above article, axis OX and OY are called axis of reference.

ii. The axis of reference, of plane figures, is generally taken as the lowest line of the figure for determining $\overline{y}$ , and left line of the figure for calculating $\overline{x}$.

iii. If the given section is symmetrical about X-X axis or Y-Y axis, then the C.G of the section will lie on the axis is symmetry.

**5.4.4. Centre of Gravity of Composition Sections.** The centre of gravity of composition sections like T-sections, I-section, L-section etc., are obtained by splitting then into rectangular components. Then equations (5.1) and (5.2) are used.

**5.4.4 Problem Based on Composition Sections**

**Problems 5.1.** Find the centre of gravity of the T- section is shown in Fig. 5.2(a).

**Sol.** The given T-section is split up into two rectangles ABCD and EFGH as shown in Fig. 5.2(b). The given T-section is symmetrical about X-Y axis. Hence the C.G. of sections will lie on this axis. The lowest line of the figure is line GF. Hence the moments of the areas are taken about this line GF, which is the axis of reference in this case.

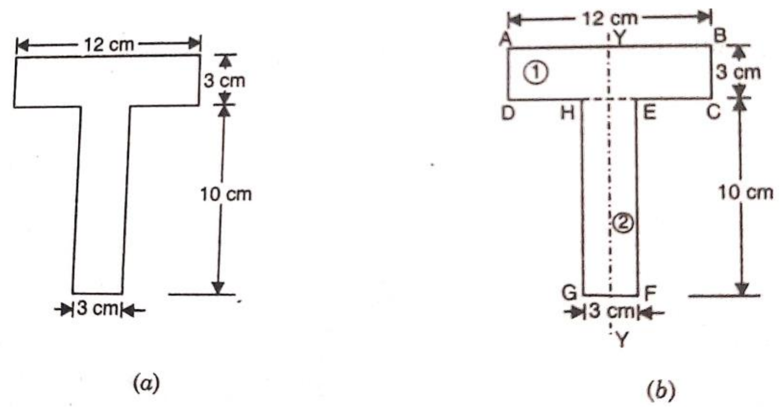


**Fig. 5.2**

Let $\overline{y}$ = The distance of the C.G. of the T-section from the bottom line GF

(which is axis of reference)

$a_1$ = Area of rectangle ABCD = 12 × 3 = 36 cm²

$y_1$ = Distance of C.G area $a_1$ from bottom line GF = $10 + \frac{3}{2}$ = 11.5 cm

$a_2$ = Area of rectangle EFGH = 10 × 3 = 30 cm²

$y_1$ = Distance of C.G area α2 from bottom line GF = $\frac{10}{2}$ = 5 cm.

Using equation (5.2), we have

$$\overline{y} = \frac{a_1 y_1 + a_2 y_2.}{\text{A}} = \frac{a_1 y_1 + a_2 y_2.}{a_1 + a_1} \qquad (\therefore \ \text{A} = a_1 + a_1)$$

$$= \frac{36 \times 11.5 + 30 \times 5}{36 + 30} = \frac{414 + 150}{66} = \mathbf{8.545\ cm.\ Ans.}$$

**Problem 5.2.** Find the centre of gravity of the I- section shown in Fig. 5.3 (a).

**Sol.** The I-section is split up into three rectangles ABCD, EFGH and JKLM as shown in Fig. 5.3 (b). The given I- section is symmetrical about X-Y axis. Hence the C.G. of the section will lie on this axis. The lowest line of the figure line is ML. Hence the moment of areas are taken about this line, which is the axis of reference.

Let $\bar{y}$ = Distance of the C.G. of the I-section from the bottom line ML

$a_1$ = Area of rectangle ABCD = 10 × 2 = 20 cm²

$y_1$ = Distance of C.G area rectangle ABCD from bottom line ML

$= 2 + 15 + \frac{2}{2} = 18$ cm

$a_2$ = Area of rectangle EFGH = 15 × 2 = 30 cm²

$y_2$ = Distance of C.G area rectangle EFGH from bottom line ML

$= 2 + \frac{15}{2} = 2 + 7.5 = 9.5$ cm

$a_3$ = Area of rectangle JKLM = 20 × 2 = 40 cm²

$y_3$ = Distance of C.G area rectangle JKLM from bottom line ML

$= \frac{2}{2} = 1.0$ cm

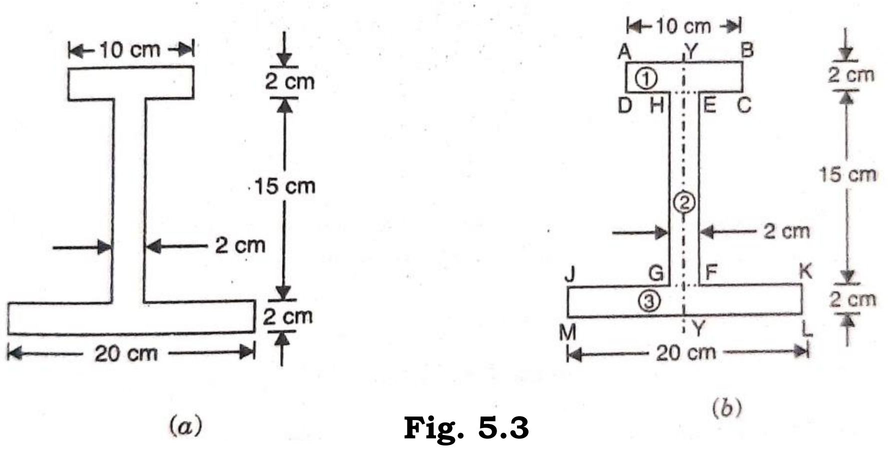


**Fig. 5.3**

Now using equation (5.2), we have

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{A}$$

$$= \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} \qquad (\therefore\ A = a_1 + a_2 + a_3)$$

$$= \frac{20 \times 18 + 30 \times 9.5 + 40 \times 1}{20 + 30 + 40}$$

$$= \frac{360 + 285 + 40}{90} = \frac{685}{90} = \mathbf{7.611\ cm.}\quad \textbf{Ans.}$$

**Problem 5.3.** *Find the centre of gravity of the L- section shown in Fig. 5.4.*

**Sol.** The given L- section is not symmetrical about any section. Hence in this case, there will be two axis of references. The lowest line of figure (i.e., line GF) will be taken as axis or reference for calculating $\overline{y}$. And the left line of the L-section (i.e., line AG) will be taken as axis of reference for calculating $\overline{x}$.

The given L- section is split up into two rectangles ABCD and DEFG, as shown in Fig. 5.4.

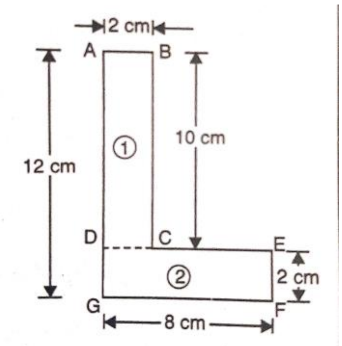


**Fig. 5.4**

To find $\overline{y}$

Let $\overline{y}$ = Distance of the C.G. of the L-section from the bottom line GF

$a_1$ = Area of rectangle ABCD = 10 × 2 = 20 cm²

$y_1$ = Distance of C.G area rectangle ABCD from bottom line GF

$= 2 + \frac{10}{2} = 2 + 5 = 7$ cm

$a_2$ = Area of rectangle DEFG = 8 × 2 = 16 cm²

$y_2$ = Distance of C.G area rectangle DEFG from bottom line GF

$= \frac{2}{2} =$ 1.0 cm.

Using equation (5.2), we have

$$\overline{y} = \frac{a_1y_1 + a_2y_2}{\text{A}}, \quad \text{where A} = a_1 + a_2$$

$$= \frac{a_1y_1 + a_2y_2}{a_1y_1 + a_2y_2} = \frac{20 \times 7 + 16 \times 4}{20 + 16} = \frac{140 + 16}{36}$$

$$= \frac{156}{36} = \frac{13}{3} = 4.33 \text{ cm.}$$

To find $\overline{x}$

Let $\overline{x}$ = Distance of the C.G. of the L-section from the bottom line AG

$\overline{x}_1$= Distance of the rectangle ABCD from left line AG

$= \frac{2}{2} = 1.0$ cm

$\overline{x}_2$ = Distance of the rectangle DEFG from left line AG

$= \frac{8}{2} = 4.0$ cm.

Using equation (5.1), we have

$$\overline{x} = \frac{a_1x_1 + a_2x_2}{A} \qquad \text{where } A = a_1 + a_2$$

$$= \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{20\times1+16\times4}{20+16} \qquad (\therefore \quad a_1 = 2\theta \text{ and } + a_2 = 16)$$

$$= \frac{20+64}{36} = \frac{84}{36} = \frac{7}{3} = 2.33\text{cm.}$$

Hence the C.G. of the L- section is at a distance of 4.33 cm from the bottom line GF and 2.33 cm from the left line AG. **Ans.**

**Problem 5.4.** Using the analytical method, determine the centre of gravity of the plane uniform lamina sown in Fig. 5.5.

**Sol.** Let $\overline{y}$ be the distance between C.G. of the lamina and the bottom line AB.

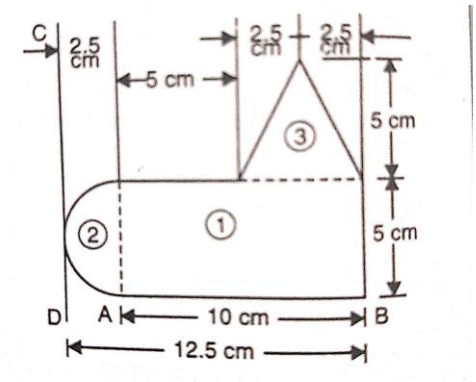


**Fig. 5.5**

Area 1

$$a_1 = 10 \times 5 = 50 \text{ cm}^2$$

$$y_1 = \frac{5}{2} = 2.5 \text{ cm}$$

Area 2

$$a_2 = \frac{\pi}{2} = r^2 = \frac{\pi}{2} \times 2.5^2 = 9.82 \text{ cm}^2$$

$$y_2 = \frac{5}{2} = 2.5 \text{ cm}$$

Area 3

$$a_3 = \frac{5\times5}{2} = 12.5 \text{ cm}^2$$

$$y_3 = 5 + \frac{5}{3} = 6.67 \text{ cm.}$$

Using the relation,

$$\overline{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{50\times2.5 + 9.82\times2.5 + 12.5\times6.67}{50 + 9.82 + 12.5}\text{cm} = \frac{232.9}{72.32} = 3.22 \text{ cm.}$$

Similarly, le $\overline{x}$ be the distance between e.g. of the lamina and the left line CD.

Area 1

$$a_1 = 50 \text{ cm}^2$$

$$x_1 = 2.5 + \frac{10}{2} = 7.5 \text{ cm}$$

Area 2

$a_2$ = 9.82 cm²

$x_2 = 2.5 - \frac{4r}{3\pi} = 2.5 - \frac{4.25}{3\Pi} = 1.44$ cm

Area 3

$a_3$ = 12.5 cm²

$x3$ = 2.5 + 5+ 2.5 = 10 cm.

Using the relation,

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{50 \times 7.5 + 9.82 \times 1.44 + 12.5 \times 10}{50 + 9.82 + 12.5} \text{ cm}$$

$$\frac{514.14}{72.32} = 7.11 \text{ cm.}$$

Hence the C.G. of the uniform lamina is at a distance of 3.22 cm from the bottom line AB and 7.11 cm from the left line CD. **Ans.**

## 5.5 CENTRE OF GRAVITY OF BODIES WITH PORTIONS REMOVED

The centre of gravity of bodies with portions removed is determined by considering the main body first as complete one and then subtracting the area of removed portion i.e., by taking the area of the area of the removed portion as negative.

**Problem 5.5.** *From a rectangular lamina ABCD 10 cm × 12 cm a rectangular whole of 3 cm × 4 cm is cut as shown in Fig. 5.6.*

*Fi*nd the e.g. of the remainder lamina.

**Sol.** The section shown in Fig. 5.6, is having a cut hole. The centre of gravity of a section with a cut hole is determined by considering the main section first as a complete one, and then subtracting the area of the cut – out hole, i.e., by taking the area of the cut-out hole as negative.

Let $\bar{y}$ is the distance between the C.G. of the section with a cut hole from the bottom line DC.

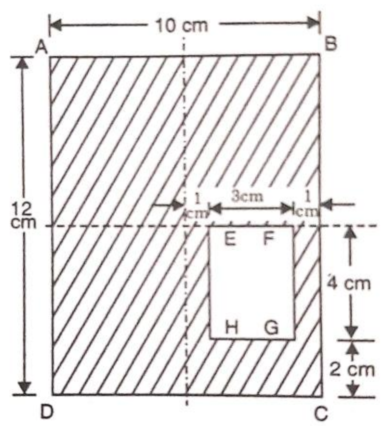


**Fig. 5.6**

$a_1$ = Area of rectangle ABCD = 10 × 12 = 120 cm²

$y_2$ = Distance of C.G. of the rectangle ABCD from bottom line DC

$= \frac{12}{2} = 6$ cm

$a_2$ = Area of cut-out hole, i.e., rectangle EFGH,

$y_2$ = Distance of C.G. of cut-out from bottom line in DC

$= 2 + \frac{4}{2} = 2 + 2 = 4$ cm.

Now using equation (5.2) and taking the area (α2) of the cut-out hole as negative, we get

$$\bar{y} = \left(\frac{a_1y_1 - a_2y_2}{\mathrm{A}}\right)^* \quad \text{where A} = a_1 - a_2$$

$$= \frac{a_1y_1 - a_2y_2}{a_1 - a_2} \quad \text{(-ve sign is taken due to cut-out hole)}$$

$$= \frac{120 \times 6 - 12 \times 4}{120 - 12} = \frac{720 - 48}{108} = 6.22 \text{ cm.}$$

To Find $\bar{x}$

Let $\bar{x}$ = Distance between the C.G. of the section with a cut hole from the left line AD

$x_1$ = Distance of the C.G. of the rectangle ABCD from the left line AD

$= \frac{10}{2} = 5$ cm

$x_2$ = Distance of the C.G. of the cut-hole from the left line AD

$= 5 + 1 + \frac{3}{2} = 7.5$ cm.

Using equation (5.1) and taking area (α2) of the cut hole as negative, we get

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} \qquad (\therefore \;\; \text{A} = a_1 - a_2)$$

$$= \frac{120 \times 5 - 12 \times 7.5}{120 - 12} = \frac{600 - 90}{108} = \frac{510}{108} = 4.72 \text{ cm.}$$

Hence the C.G. of the section with a cut hole will be at distance of 6.22 cm from line DC and 4.72 cm from the line AD. **Ans.**

**5.5.1. Problems of Finding Centre of Gravity of Area by Integration Method**

**Problem.5.6.** Determine the co – ordinates of the C.G. of the area OAB sown in Fig. 5.7, if the curve OB represents the equation of a parabola, given by

$$y = kx^2$$

in which OA = 6 units

and AB = 4 units.

**Sol.** The equation of parabola is $y = kx^2$ ...(i)

First determine the value of constant $k$. The point B is lying on the curve and having co–ordinates

$$x = 6 \text{ and } y = 4$$

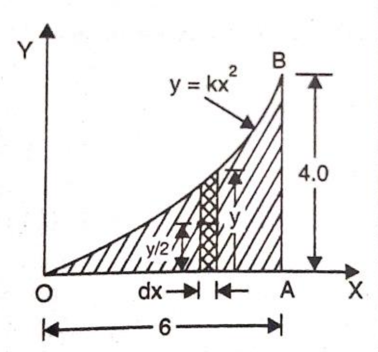


**Fig. 5.7**

Substituting these values of equation (i), we get

$$4 = k \times 6^2 = 36\ k$$

$$\therefore \quad k = \frac{4}{36} = \frac{1}{9}$$

Substituting the value of $k$ equation (i), we get

$$y = \frac{1}{9} x^2 \quad \text{...(ii)}$$

or $$x^2 = 9\,y$$

or $$x = 3\sqrt{y} \quad \text{...(iii)}$$

Consider a strip of height y and width dx as shown in Fig. 5.7. the area dA of the strip is give by

$$dA = y \times dx$$

The co-ordinates of the C.G. of this area dA are x and $\frac{y}{2}$

$\therefore$ Distance of C.G. of area dA from y-axis = $x$

and Distance of C.G. of area dA from x-axis = $\frac{y}{2}$

$$\therefore \quad x* = x \text{ and } y* = \frac{y}{2}$$

Let $\bar{x}$ = Distance of C.G. of total area OAB from axis OY

$\bar{y}$ = Distance of C.G. of total area OAB from axis OX.

Using equation [5.2 (A)], we get

$$\bar{x} = \frac{\int x^* dA}{\int dA}$$

($\therefore$ $dA = y \times dx, x^* = x$)

But $y = \frac{x^2}{2}$ from equation (ii),

$$\therefore \qquad \bar{x} = \frac{\int_0^6 x \times \frac{x^2}{9} \times dx}{\int_0^6 \frac{x^2}{9}\, dx} = \frac{\frac{1}{9}\int_0^6 x^3\, dx}{\frac{1}{9}\int_0^6 x^2\, dx}$$

$$= \frac{\int_0^6 x^3\, dx}{\int_0^6 x^2\, dx} = \frac{\left[\frac{x^4}{4}\right]_0^6}{\left[\frac{x^3}{3}\right]_0^6} = \frac{\frac{1}{4} \times 6^4}{\frac{1}{3} \times 6^3}$$

$$= \frac{1}{4} \times \frac{3}{1} \times 6 = 4.5. \quad \text{Ans.}$$

Using equation [5.2 (B)], we get

$$\bar{y} = \frac{\int y^* dA}{\int dA}$$

Where $y^*$ = Distance of C.G. of area $dA$ from $x$-axis

$= \frac{y}{2}$ (here)

$$dA = y dx$$

$$\therefore \qquad \int y^* dA = \int \frac{y}{2} \times dA = \int_0^6 \frac{y}{2} \times y dx \times \int_0^6 \frac{y^2}{2}\, dx$$

$$= \frac{1}{2}\int_0^6 \frac{x^2}{81}\, dx = \frac{1}{2} \times \frac{1}{81}\int_0^6 x^4\, dx \; \frac{1}{2} \times \frac{1}{81}\left[\frac{x^5}{5}\right]_0^6$$

$$= \frac{1}{2} \times \frac{1}{81} \times \frac{6^5}{5} = \frac{6^5}{810}$$

**Problem 5.7.** determine the co-ordinates of the C.G. of the shaded area between the parabola = $\frac{x^2}{4}$ and the straight line $y = x$ as shown in Fig. 5.8.

**Sol.** The equation of parabola and straight line are

$$y = \frac{x^2}{4} \qquad \ldots\text{(i)}$$

$$y = x \qquad \ldots\text{(ii)}$$

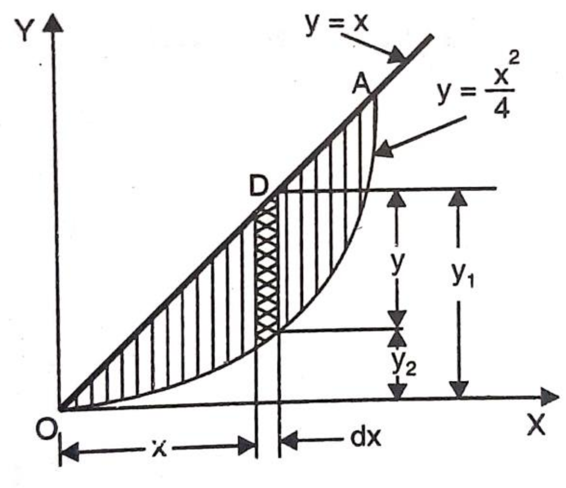


**Fig. 5.8**

The point A is lying on the straight line as well as on the given parabola. Hence both the above equations hold good for point A. Let the co-ordinates of point A. Let the co-ordinates of point A are $x, y$.

Substituting the value of y from equation (ii),

$$x = \frac{x^2}{4} \quad \text{or} \quad 4 = \frac{x^2}{x} = x$$

Substituting the value of $x = 4$ from equation (ii),

$$y = 4$$

Hence the co-ordinates of point A are 4, 4.

Now divide the shaded area into large small areas each of height y and width d$x$ as shown in Fig. 5.8. then area dA of the strip is given by

$$dA = y dx = (y_1 - y_2) dx \qquad \text{...(iii)}$$

where $y_1$ = co-ordinate of point D which lies on the straight line OA

$y_2$ = co-ordinate of point E which lies on the parabola OA.

The horizontal co-ordinates of the points D and E are same.

The values of $y_1$ and $y_2$ can be obtained in terms of $x$ from equations (ii) and (i),

$$y_1 = x \quad \text{and} \quad y_2 = \frac{x^2}{4}$$

Substituting these values in equation (iii),

$$dA = \left(x - \frac{x^2}{4}\right) dx \qquad \text{...(iv)}$$

The distance of the C.G. for the area dA from y-axis is given by,

$$x* = x$$

And the distance of the C.G. of the area dA from x-axis is given by,

$$y^* = y_2 + \frac{y}{2} = y_2 + \frac{y1-y2}{2} \qquad (\therefore\ y = y_1 - y_2)$$

$$= \frac{2y2+y1-y2}{2} = \frac{y1+y2}{2}$$

$$= \frac{x+\frac{x^2}{4}}{2} \qquad (\therefore y_1 = x \text{ and } y_2 = \frac{x^2}{4})$$

$$= \frac{1}{2}\left(x + \frac{x^2}{4}\right) \qquad \ldots(v)$$

Now let $\overline{x}$ = Distance of C.G. of shaded area of Fig. 5.8 from y-axis

$\overline{y}$ = Distance of C.G. of shaded area of Fig. 5.8 from x-axis.

Now using equation [5.2 (A)],

$$\overline{x} = \frac{\int x*dA}{\int dA}, \qquad \text{where } x *= x$$

$$\text{dA} = \left(x - \frac{x^2}{4}\right)\text{d}x \qquad \text{[See equation (iv)]}$$

$$\therefore \qquad \int x^*\text{dA} = \int_0^4 x\left(x - \frac{x^2}{4}\right)\text{d}x \qquad (\therefore\ x \text{ varies from 0 to 4})$$

$$= \int_0^4 \left(x - \frac{x^2}{4}\right)\text{d}x = \left[\frac{x^3}{3} - \frac{x^4}{4\times 4}\right]_0^4$$

$$= \frac{x^3}{3} - \frac{x^4}{4\times 4} = \frac{64}{3} - 16$$

$$= \frac{64-48}{3} = \frac{16}{3}$$

and

$$\int dA = \int_0^4 \left(x - \frac{x^2}{4}\right)dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3\times 4}\right]_0^4 = \frac{4^2}{2} - \frac{4^3}{3\times 4}$$

$$= \frac{16}{2} - \frac{16}{3} = \frac{48-32}{6} = \frac{16}{6}$$

$$\therefore \qquad \overline{x} = \frac{\int x^*dA}{\int dA} = \frac{\frac{16}{3}}{\frac{16}{3}} = \frac{16}{3} \times \frac{6}{16} = \mathbf{2 Ans.}$$

Now using equation [5.2 (B)], $\overline{y} = \frac{\int y*dA}{\int dA}$

where $$y^* = \frac{1}{2}\left(x + \frac{x^2}{4}\right) \text{.....} \qquad \text{[From equation (v)]}$$

$$dA = \left(x - \frac{x^2}{4}\right)\mathrm{d}x$$

$$\int x^*\mathrm{dA} = \int_0^4 \frac{1}{2}\left(x - \frac{x^2}{4}\right)\left(x - \frac{x^2}{4}\right)\mathrm{d}x$$

$$= \frac{1}{2}\int_0^4 \left(x^2 - \frac{x^4}{16}\right) dx = \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^5}{5 \times 16}\right]_0^4$$

$$= \frac{1}{2}\left[\frac{4^3}{3} - \frac{4^5}{5 \times 16}\right] = \frac{1}{2}\left[\frac{64}{3} - \frac{64}{5}\right]$$

$$= \frac{64}{2}\left[\frac{1}{3} - \frac{1}{5}\right] = 32\left(\frac{5-3}{15}\right)$$

$$32 \times \frac{2}{15} = \frac{64}{15}$$

and $$\int dA = \frac{16}{6}$$

$\therefore$ $$\overline{x} = \frac{\int y^* dA}{\int dA} = \frac{\frac{64}{15}}{\frac{16}{6}} = \frac{64}{15} \times \frac{6}{16} = \mathbf{\frac{8}{5}} \textbf{ Ans.}$$

**Problem 5.8.** Determine the centre of gravity of the area of the circular sector OAB of radius R and central angle α as shown in Fig. 5.9.

**Sol.** The given area is symmetrical about *x*-axis. Hence the C.G. of the area will lie on x-axis. This means $\overline{y} = 0$. To find $\overline{x}$, the moment of small areas are to be taken along y-axis. Divide the area OAB into a large number Of triangular element each of altitude R and base Rd θ as shown in Fig. 5.10. such triangular element is shown by OCD in which altitude OC = R and base CD = Rd θ. The area dA of this triangular element is given by,

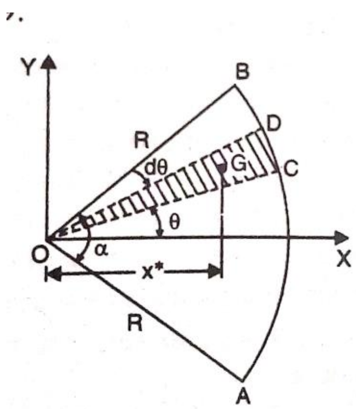


**Fig. 5.9**

$$dA = \frac{OC \times CD}{2} = \frac{R \times R d\theta}{2}$$

$$= \frac{R^2 d\theta}{2}$$

The C.G. of this triangular element is at G

where $$OG = \frac{2}{3} \times OC = \frac{2}{3} \times R$$

The distance of C.G. area and $d$A from y-axis is given by,

$$x^* = OG \times \cos\theta = \frac{2}{3} \times R \times \cos\theta$$

Now using equation [5.2 (A)],

$$\bar{x} = \frac{\int x^* dA}{\int dA} = \frac{2\int_0^{\alpha/2}\left(\frac{2}{3}R\cos\theta\right)\left(\frac{R^2\,d\theta}{3}\right)}{2\int_0^{\alpha/2}\frac{R^2}{2}\,d\theta}$$

$$= \frac{\frac{R^2}{2}\int_0^{\alpha/2}\cos\theta\,d\theta}{\frac{R^2}{2}\int_0^{\alpha/2}d\theta} = \frac{2R}{3}\frac{[\sin\theta]_0^{\alpha/2}}{[\theta]_0^{\alpha/2}}$$

$$= \frac{2R}{3}\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} = \frac{4R}{3\alpha}\sin\frac{\alpha}{2} \quad \textbf{Ans.}$$

The area OAB is symmetrical about the x-axis, hence

$$\bar{y} = \mathbf{0}. \quad \textbf{Ans.}$$

For a semi-circle, α = π = 180°, hence

$$\bar{x} = \frac{4R}{3\alpha}\sin\left(\frac{\pi}{2}\right)$$

$$= \frac{4R}{3\times\pi}\sin\left(\frac{180}{2}\right) = \frac{4R}{3\pi}. \quad \text{Ans.}$$

**Problem 5.9.** Determine the centre of gravity of a semi-circle of radius R as shown in Fig. 5.10.

**Sol.** This problem can also be solved by the method given in problem 5.8. The following other methods can also be used. Due to symmetry, $\bar{x}$ = 0. The area AOB is symmetrical about the y-axis. Hence $\bar{x}$ = 0. The value of $\bar{y}$ is obtained by taking the moments of small areas and total area about x-axis.

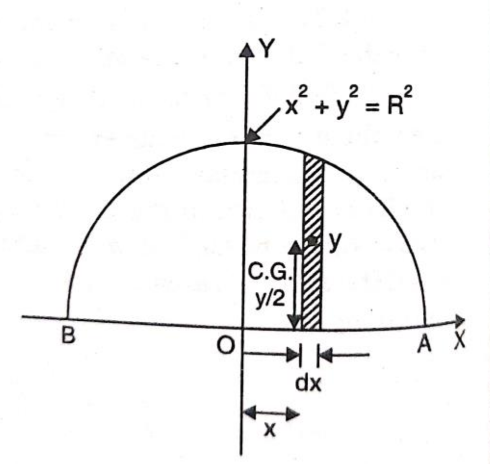


**Fig. 5.10**

**1. Considering the strip parallel to y-axis**

Area of strip, dA = y, d$x$

The distance of the C.G of the area dA from $x$-axis is equal to $\frac{y}{2}$

Moment of area dA about x-axis

$$= dA.\ \frac{y}{2}$$
$$= \frac{y}{2}.dA$$
$$= \frac{y}{2}.ydx$$
$$= \frac{y^2}{2}.dx$$

Moment of total area A about x-axis is obtained by integrating the above equation.

∴ Moment of total area A about x-axis

$$=\int \frac{y^2}{2}.dx$$

$$=\int_{-R}^{R} \frac{y^2}{2} \qquad (\therefore\ x \text{ varies from } -R \text{ to } R)$$

But equation of semi-circle is

$$x^2 + y^2 = R^2 \text{ or } \qquad y^2 = R^2 - x^2$$

Substituting this value of $y^2$ in the above equation, we get

Moment of total area A about x-axis

$$=\int_{-R}^{R} \frac{(R^2 - x^2)}{2}\,dx$$

$$=\frac{1}{2}\left[R^2.x - \frac{x3}{3}\right]_{-R}^{R}$$

$$=\frac{1}{2}\left(R^2.R-\frac{R3}{3}\right) -$$

=............

=...........

=............

Let $\overline{y}$ = Distance of C.G of the total area of semi-circle from x-axis.

The total area of semi-circle is also equal to $\frac{\pi R^2}{2}$

$\therefore$ moment of total area A about x-axis

$$=\overline{y}\times\frac{\pi R^2}{2} \qquad \ldots\text{(ii)}$$

Equating the two values given by equations (i) and (ii), we get

$$=\overline{y}\times\frac{\pi R^2}{3}=\frac{2R3}{3}$$

$\therefore$ $\qquad \overline{y}=\frac{2R3}{3}\times\frac{2}{\pi R^2}=\frac{4R}{3\pi}.$ Ans.

**2.Considering the strip parallel to $x$-axis**

Area of strip, dA= $2x$,d$y$
The distance of the C.G of the area from

x-axis is y

$\therefore$ Moment of total area about x-axis

$$= y.dA$$

$$= y.2xdy$$

$$= 2xy\ dy$$

But, we know $x^2+y^2=R^2$

$\therefore$ $\qquad x^2=R^2-y^2$

or $\qquad x=\sqrt{R^2-y^2}$

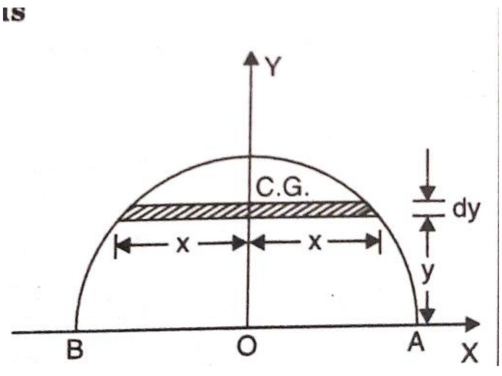


**Fig. 5.10(a)**

Substituting the above value of $x$ in equation (i), we get

Moment of area dA about $x$-axis,

$$= 2\sqrt{R^2-y^2},\ y.dy$$

Moment of total area A about $x$-axis will be obtained by integrating the above equation from O to R.

$\therefore$ Moment of area A about $x$-axis

$$= \int_0^R 2\sqrt{R^2 - y^2}, y.dy \qquad (\therefore\ y \text{ varies from O to R})$$

$$= -\int_0^R {}^2\sqrt{R^2 - y^2}\,.(-2\,y)dy \;=\; -\left[\frac{(R^2-y^2)^{3/2}}{3/2}\right]_0^R$$

$$= -\frac{2}{3}\,[0\text{-}R^2] = \frac{2R^3}{3} \qquad \ldots\text{(i)}$$

Also the moment of total area A about $x$-axis = A × $\overline{y}$

where A = Total of semi-circle = $\frac{\pi R^2}{2}$

$\overline{y}$ = Distance of C.G of area A from $x$-axis

$\therefore$ Moment of total area A about $x$-axis = $\frac{\pi R^2}{2} \times \overline{y}$ ...(ii)

Equating the two values given by equations (i) and (ii),

$$\frac{\pi R^2}{2} \times \overline{y} = \frac{2R3}{3}$$

or $\overline{y} = \frac{2R^3}{3} \times \frac{2}{\pi R^2} = \frac{4R}{3\pi}$. Ans.

**Problem 5.10.** To determine the centre of gravity of the area shown in Fig. 5.10(b) given by $\frac{x^2}{\alpha^2} + \frac{y^2}{b^2} = 1$.

**Sol.** Consider a small strip of thickness $dx$ parallel to y-axis at a distance of $x$ from the y-axis.

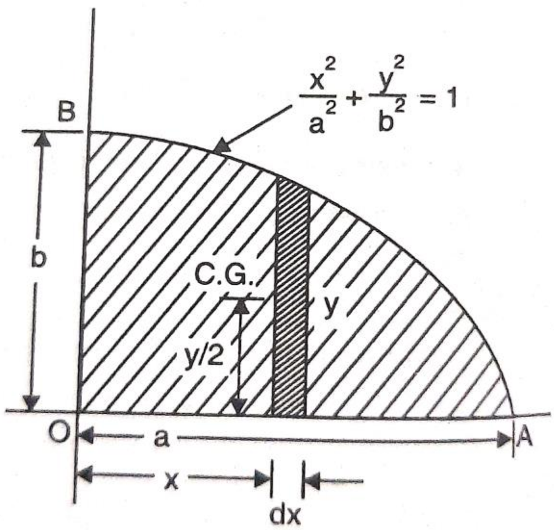


**Fig. 5.10(b)**

Area of strip, dA= $y$.d$x$

The C.G of area dA is at a distance of $\frac{y}{2}$ from x-axis.

Moment of total area dA about $x$-axis

$$= \frac{y}{2}.\text{dA}$$

$$= \frac{y}{2}\ y\text{d}x$$

$$(\therefore\ \ \text{dA} = y.\text{d}x)$$

$$= \frac{y}{2}.\text{d}x$$

$\therefore$Moment of total area about $x$-axis

$$= \int_0^{\alpha} \frac{y^2}{2}.\text{d}x \qquad (\therefore\ \ x \text{ varies from O to } \alpha)\ldots\text{(i)}$$

Let us substituting the value of $y^2$ in terms of $x$,

The given equation is $\frac{x^2}{\alpha^2} + \frac{y^2}{b^2}$

or $$\frac{y^2}{b^2} = 1 - \frac{x^2}{\alpha^2} = \frac{\alpha^2 - x^2}{\alpha^2}$$

or $$y^2 = \frac{b^2}{\alpha^2}(\alpha^2 - x^2) \qquad \ldots\text{(ii)}$$

Substituting the value of $y^2$ in equation (i), we get

Moment of total area about $x$-axis

$$= \frac{1}{2}\int_0^{\alpha} \frac{b^2}{\alpha^2}(\alpha^2 - x^2)\ dx = \frac{b^2}{\alpha^2}\left[a^2x - \frac{x3}{3}\right]_0^{a}$$

$$= \frac{b^2}{2\alpha^2}\left[a^3 - \frac{\alpha 3}{3}\right] = \frac{b^2}{2\alpha^2} \times \frac{2a^3}{3} = \frac{\alpha b^2}{3} \qquad \ldots\text{(iii)}$$

The total area A of the given figure is given by

$$\text{A} = \int \text{dA} = \int y.\text{d}x \qquad \ldots\text{(iv)}$$

From equation (ii), $\text{y} = \left[\frac{b^2}{\alpha 2}(\alpha^2 - x^2)\right] .. \frac{b}{\alpha}(\alpha^2 - x^2)^{1/2}$ ...(v)

Now equation(iv) is $\text{A} = \int_0^{\alpha} \frac{b^2}{\alpha^2}(\alpha^2 - x^2)^{1/3}\ \text{d}x$

$$= \frac{b}{\alpha}\left[\int_0^{\alpha}(\alpha^2 - x^2)^{1/2}.dx\right]^* = \frac{b}{\alpha}\left[\frac{\pi\alpha^2}{4}\right]^*$$

$$= \frac{\pi.\alpha b}{4} \qquad \left(\int \sqrt{\alpha^2 - x^2} dx \frac{\pi\alpha^4}{4}\right) \qquad \ldots\text{(vi)}$$

Let $\overline{y}$ = the distance of C.G of the total area A from x-axis.

Then moment of total area A about $x$-axis

$= \text{A} \times \overline{y}$

$= \frac{\pi\alpha b}{4}.\ \overline{y}$ ...(vii)

The equations (iii) and (vi) give the moment of total area about x-axis. Hence equating these equations, we get

$\frac{\pi\alpha b}{4}.\ \overline{y} = \frac{\alpha b^2}{3}$

$\therefore \quad \overline{y} = \frac{\alpha b^2}{3}.\ \frac{4}{\pi\alpha b} = \frac{4b}{3\pi}.$ Ans.

To find $\overline{x}$ , take the moment of small area dA about y-axis.

The C.G. of area dA is at a distance of x from y-axis.

$\therefore$ Moment of area dA about y-axis = $x.dA$

$= x.y.dx$ $(\therefore\ dA = ydx)$

Moment of total area A about y-axis is obtained by integration

Moment of area A about y-axis

$= \int_0^\alpha x.y.dx$ ($\therefore$ $x$ varies from O to α)

$= \int_0^\alpha x.\frac{b}{\alpha}(\alpha^2 - x^2)^{1/2}\ .dx$

$\left[\therefore y = \frac{b}{\alpha}(\alpha^2 - x^2)^{1/2} \text{from equation(v)}\right]$

$= \frac{b}{\alpha}\int_0^\alpha x.\frac{b}{\alpha}(\alpha^2 - x^2)^{1/2}.dx = \frac{b}{\alpha}\int_0^\alpha \frac{(-2)}{(-2)}.x\,(\alpha^2 - x^2)^{1/2}dx$

$= \frac{b}{-2\alpha}\left[\frac{(\alpha^2-x^2)^{1/2}}{3/2}\right].. = \frac{-b}{3\alpha}[0 - a^3] = \frac{b\alpha^2}{3}$ ...(viii)

Also the moment of total area A about y-axis.

$= \text{A} \times \overline{x}$ ...(ix)

Where $\overline{x}$ = Distance of C.G. of total area A from y-axis.

Equating the two values given by equations (viii) and (ix),

$\text{A} \times \overline{x} = \frac{b\alpha^2}{3}$

$\therefore \quad \overline{x} = \frac{b\alpha^2}{3A} = \frac{b\alpha^2}{3\times\frac{\pi\alpha b}{4}}$ $\left[\therefore \text{A} = \frac{\pi\alpha b}{4} \text{See equation (vi)}\right]$

$\frac{4\alpha}{3A}$. Ans.

The co-ordinates of the C.G. of given area are

$$\overline{x} = \frac{4\alpha}{3A} \text{ and } \overline{y} = \frac{4b}{3\pi}$$

## STUDENT ACTIVITY

1. Drive an expression for the entre of gravity of a plane area using method of moments.

2. What do you understand by axes of references?

# SUMMARY

1. The point through which the whole weight of the body acts, is known as centre of gravity.
2. The point at which the total area of a plane figure is assumed to be concentrated is known is centroid of that area. The centroid and centre of gravity are at the same point.
3. The centre of gravity of a uniform rod lines at its middle point.
4. The C.G of a triangle lies at a point where the three medians of a triangle meet.
5. The C.G of a parallelogram or a rectangle is at a point where its diagonal meet each other.
6. The C.G of a circle lies its centre.
7. The C.G of as a body consisting of different areas is given by

$$\overline{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \ldots\ldots\ldots\ldots}{a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots} \quad \text{and} \quad \overline{y} = \frac{a_1y_1 + a_2y_2 + a_3y_2 + \ldots\ldots\ldots\ldots}{a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots}$$

where $\overline{x}$ and $\overline{y}$ = Co-ordinate of the C.G of the body from axis of reference

$a_1, a_2, a_3, \ldots\ldots$ = Different areas of the sections of the body

$x_1, x_2, x_3, \ldots\ldots$ = Different of the C.G. of the areas $a_1, a_2, a_3, \ldots\ldots$ from Y-axis.

$y_1, y_2, y_3 \ldots\ldots\ldots$= Different of the C.G. of the areas $a_1, a_2, a_3, \ldots\ldots$ from X-axis.

8. If a given section is symmetrical about X-X axis or Y-Y axis, the C.G. of the section will lie on the axis symmetry.
9. The C.G. of an area by integration method is given by

$$\overline{x} = \frac{\int x^* dA}{\int dA} \text{ and } \overline{y} = \frac{\int y^* dA}{\int dA}$$

Where $x*$ = Distance of C.G. of area dA from y-axis

$y*$ = Distance of C.G. of area dA from x-axis.

10. The C.G. of a straight or curved line is given by

$$\overline{x} = \frac{\int x^* dL}{\int dL} \text{ and } \overline{y} = \frac{\int y^* dL}{\int dL}$$

# TEST YOURSELF

**(A) Theoretical Problems**

1. Define centre of gravity and centroid.
2. Derive an expression for the centre of gravity of a plane area using method of moments.
3. What do you understand by axes of reference?

## (B) Numerical Problems

1. Find the centre of gravity of the T-section shown in Fig.5.11.

[**Ans**.8.272 cm]

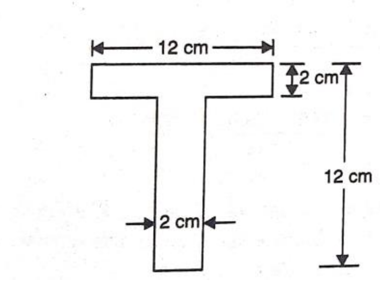


**Fig. 5.11**

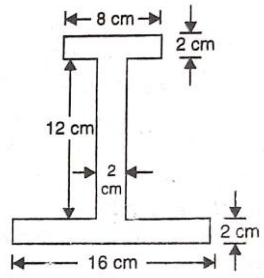


**Fig. 5.12**

2. Find the centre of gravity of the I-section shown in Fig.5.12.

[**Ans.**6.44 cm]

3. Find the centre of gravity of the I-section shown in Fig.5.13.

[**Ans.** $\bar{x}$=1.857, and $\bar{y}$=3.857]

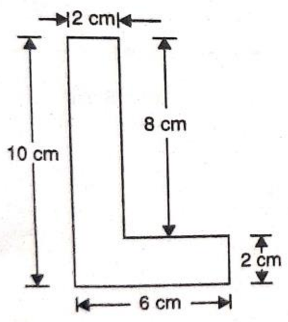


**Fig. 5.13**

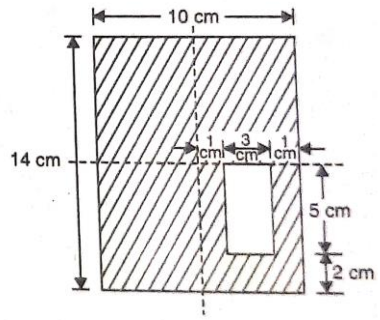


**Fig. 5.14**

4. From a rectangular lamina *ABCD* 10cm×14cm a rectangular hole of 3cm×5cm is cut as shown in Fig.5.14. Find the centre of gravity of the remainder lamina.

[**Ans.** $\bar{x}$=4.7cm, and $\bar{y}$=6.444cm]

5. Locate the C.G of the area shown in Fig.5.15 with respect to co-ordinate axes. All dimensions are in mm.

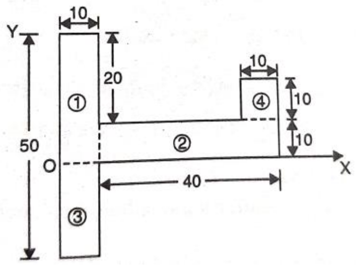


**Fig. 5.15**

**[Hint,**

$a_1$=10 × 30 =300 mm², $x^1$ = 5 mm, $y_1$=15

$a_2$= 40 × 10 = 400 mm², $x_2$ = 10 + 20 =13 mm, $y_2$ = 5 mm

$a_3$= 10× 20 = 200 mm², $x_3$ = 5mm, $y_3$ = -10 mm

$a_4$ = 10 × 10 = 100 mm², $x_4$= 45 mm,

$y_4$ = 10 + 5 mm

$$\overline{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4}{a_1+ a_2 + a_3+a_4} = \frac{1500 + 12000 + 1000 + 4500}{1000}$$

= 1.5 + 12 + 1 + 4.5 = 19 mm. **Ans.**

$$\overline{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4}{a_1+ a_2 + a_3+a_4} = \frac{4500+2000-2000+1500}{1000}$$

= 4.5 + 2 – 2 + 1.5 = 6mm. **Ans.**

**6.** A thin homogenous wire is bent into a triangular shape ABC such that AB= 240mm, BC=260mm and AC=100mm. Locate the C.G. of the wire with respect to co-ordinate axes. Angle at A is right angle.

**[Hint,** First determine angles α and β. Use sine rule

$$\frac{\text{BC}}{\sin 90°}.= \frac{\text{AC}}{\sin\alpha}.= \frac{\text{AB}}{\sin\beta}.$$

∴ $\sin\alpha = \frac{\text{AC} \times \sin 90°}{\text{BC}} = \frac{100}{260}.$

∴ α = 22.62°

Also $\sin\beta = \frac{\text{AB}}{\text{BC}} \times \sin 90° = \frac{240}{260}.$

∴ β = 67.38°

Using equation [5.2(C)] and [5.2(D)]

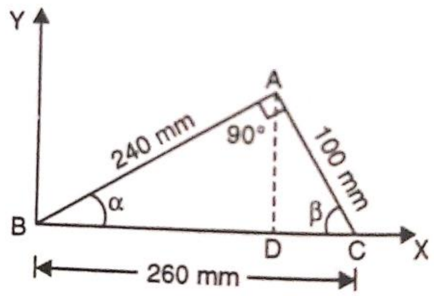


**Fig. 5.16**

$\overline{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1+ L_2 + L_3}$, where $L_1$ = AB =240,

$x_1$ = distance of C.G. of AB from y-axis

= $\frac{240}{2}$ × cos α = 120 × cos 22.62°= 110.77mm

$L_2$= BC = 260 mm, $x_2$ = distance of C.G. of BC from y-axis = 130

$L_3$= AC = 100 mm, $x_3$ = distance of C.G. of AC from y-axis

= BD + $\frac{100}{2}$ cos β = 240 cos α + 50 cos β

= 240 × cos 22.62° + 50 cos 67.38° = 240.77

$$\overline{x} = \frac{240 \times 110.77 + 260 \times 130 + 100 \times 240.77}{240 + 260 + 100} = 140.77\text{mm}.$$ **Ans.**

$$\overline{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3}{L_1 + L_2 + L_3}$$

where $y_1 = \frac{240}{2}$ sin α = 120 × sin 22.62°= 46.154

$y_2 = 0$, $y_3 = \frac{100}{2}$ sin β = 50 sin 67.38° = 48.154

∴ $$\overline{y} = \frac{240\times46.154+260\times0+100\times46.154}{600}$$

= 26.154mm. **Ans.**

**7.** Determine the C.G. of the uniform plane lamina shown in Fig. 5.17. all dimensions are in cm.

**[Hint.** The figure is symmetrical about Y-Y axis,

$$\overline{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4}{a_1 + a_2 + a_3 + a_4}$$

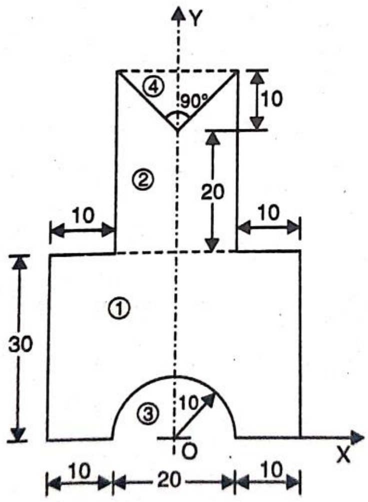


**Fig. 5.17**

Where

$a_1$ = 40 × 30 = 1200cm², $y_1 = \frac{30}{2}$ = 15cm,

$a_2$= 30 × 20 = 600cm², $y_2 = 30 + \frac{30}{2}$ = 45cm

$a_3 = \frac{\pi\times10^2}{2} = 50\pi$, $y_3 = \frac{4r}{3\pi} = \frac{4\times10}{3\pi} = \frac{40}{3\pi}$

$a_4 = \frac{20\times10}{2}$ = - 100, $y_4 = 60 - \frac{10}{3} = \frac{170}{3}$

∴ $$\overline{y} = \frac{1200\times15 + 600\times45 - 50\pi\times\frac{40}{3\pi} - 100\times\frac{170}{3}}{1200 + 600 - 50\pi - 100}$$

$$= \frac{18000 + 27000 - 666.7 - 5666.7}{1700 - 50\pi} = \frac{38666.6}{1542.92}$$

= 25.06 cm from origin 0. **Ans.**

# 6 LAWS OF MOTION

**LEARNING OBJECTIVES**

- Concept of Momentum
- Newton’s Laws of Motion
- Newton’s First Law of Motion
- Derivation of Force Equation from Second Law of Motion
- Numerical Problems on Second Law of Motion
- Motion of Two Bodies Tied by a String
- Law of Conservation of Momentum

## 6.1 concept of momentum

The product of the mass of a body and its velocity is known as momentum of the body. Momentum is a vector quantity. Mathematically, momentum is given by

$$\text{Momentum} = \text{Mass} \times \text{Velocity}.$$

## 6.2 NEWTON’S LAWS OF MOTION

When a body is at rest or moving in a straight line or rotating about an axis, the body obeys certain laws of motion. These laws are called Newton's laws of motion. There are three laws of motion.

**6.2.1. First Law.** It states that a body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external force to change that state.

**6.2.2. Second Law.** It states that the rate of change of momentum of a body is proportional to the external force applied on the body and takes place in the direction of the force.

**6.2.3. Third Law.** It states that to every action, there is always an equal and opposite reaction.

Before discussing Newton's laws of motion, let us define certain terms like mass and weight.

**6.2.4. Mass.** The quantity of matter contained in a body is known as mans of the body. Mass is a scalar quantity. In C.G.S. units, the mans is expressed in gram (gm) whereas in S.I. units the mass is expressed in kilogram (kg).

**6.2.5. Weight.** Weight of a body is defined as the force, by which the body is attracted towards the centre of the earth. Mathematically weight of a body is given by

$$\text{Weight} = \text{mass} \times \text{acceleration due to gravity} = \text{mass} \times g$$

If mass is taken in kilogram (kg) and acceleration due to gravity in metre per second square ($m/s^2$), then weight is expressed in newton (N). But if mass is taken in gram (gm) and acceleration due to gravity in centimetre per second square ($cm/s^2$), then weight is expressed in dyne. The relation between newton (N) and dyne is given as

$$\text{One Newton} = 10^5 \text{ dyne.}$$

## 6.3 NEWTON'S FIRST LAW OF MOTION

It consists of two parts. First part states that a body continues in its state of rest unless it is compelled by an external force to change that state. A book lying on a table remains at rest, unless it is lifted by some external force.

Second part states that a body continues in its state of uniform motion in a straight line unless it is compelled by an external force to change that state. In actual practice, we see that when a body is moving with a uniform velocity in a straight line, the body does not continue in its state of uniform motion but comes to rest after some time. This is due to frictional force acting on the body. For an ideal case (i.e., when there is no frictional force noting on the body), the body will continue to move with uniform velocity in a straight line, unless compelled by an external force to change that state.

## 6.4 DERIVATION OF FORCE EQUATION FROM SECOND LAW OF MOTION

Second law of motion enables us to measure force. Let u body of mass '$m$' is moving with a velocity '$u$' along a straight line. It is acted upon by a force $F$ and the velocity of the body becomes $v$ in time $t$. Then we have

$u$ = Initial velocity of the body,

$v$ = Final velocity of the body,

$m$ = MASS of the body,

$a$ = Uniform linear acceleration,

$F$ = Force acting on the body, which changes the velocity $u$ to $v$ in time $t$,

$t$ = Time in second to change the velocity from to $u$ to $v$. Initial momntum of the body.

= Mass × initial velocity = $m \times u$

Final momentum of the body = $m \times v$.

∴ Change of momentum

$$= \text{Final momentum - Initial momentum} = mv - mu = m(v - u)$$

Rate of change of momentum = $\dfrac{\text{Change in momentum}}{\text{time}} = \dfrac{m(v-u)}{t}$ ...(i)

But we know that,

$$\frac{v-u}{t} = a \qquad \text{(i.e., linear acceleration)}$$

Substituting the value of $\left(\dfrac{v-u}{t}\right)$ in equation (i), we get

Rate of change of momentum = $m \times a$.

But according to Newton's second law of motion, the rate of change of momentum is directly proportional to the external force acting on the body.

∴ $F \propto m \times a$ or $F = k \times m \times a$ ...(ii)

where k is a constant of proportionality.

In equation (ii), $k$ and $m$ (mass of a body) are constants for a given body and hence force acting on a body is proportional to the acceleration produced by the force. This means that for a given body, greater force products greater acceleration while a smaller force produces smaller acceleration. The acceleration produced will be zero if no force is applied on the body.

Two important conclusions are drawn from the first two Newton's laws of motion:

(i) There will be no acceleration, if no external force is applied on the body. This means the body will continue in its state of existing uniform motion in a straight line.

(ii) Force applied on the body is proportional to the product of mass of the body and the acceleration produced by the force.

**6.4.1. Unit of Force.** Let us first define a 'unit force'. A unit force can be suit- ably defined so as to make the value of k in equation (ii) equal to one. A unit force (i.e., Force = 1.0) is that which produces unit acceleration on an unit mass. Then by substituting $F = 1.0$, $m = 1.0$ and $a = 1.0$ in equation (ii) (i.e., $F = k \times m \times a$), we get

$$1 = k \times 1 \times 1 \qquad \text{or} \qquad k = 1$$

Substituting the value of $k = 1$, in equation (ii), we get

$$F = m \times a \qquad \ldots(6.2)$$

(i) If mass ($m$) = 1 kg and acceleration produced ($a$) = 1 m/s$^2$ the unit of force is known as newton (which is written as N). Thus newton is defined as that force which acts on a body of mass one kg and produces an acceleration of 1 m/s$^2$ in the direction of force. Newton is the unit of force in S.I. system.

$$\therefore \qquad 1\ \text{N} = 1\ \text{kg} \times 1\ \text{m/s}^2 \times 1\ \text{kg- m/s}^2.$$

(ii) If mass ($m$) = 1 gm and acceleration ($a$) = 1 cm/s$^2$, then unit of force is known 'dyne'. Thus a dyne may be defined as that force which acts on a body of mass one gm and produce an acceleration of 1cm/s$^2$. Dyne is the unit of force in C.G.S. system.

$$\therefore \qquad 1\ \text{dyne} = 1\ \text{gm} \times 1\ \text{m/s}^2 = \text{gm cm/s}^2.$$

By definition

$$1\ \text{N} = 1\ (\text{kg}) \times 1\ \text{m/s}^2 = 1 \times 1000\ (\text{gm}) \times 1 \times 100\ (\text{cm/s}^2)$$

$$= 10^5\ (\text{gm cm/s}^2)$$

$$= 10^5\ \text{dyne} \qquad (\because 1\ \text{dyne} = 1\ \text{gm} \times 1\ \text{cm/s}^2 = \text{gm cm/s}^2)$$

**Note.** (i) The body will have acceleration if the external force is acting on the body in the direction of motion of the body.

(ii) The body will have retardation if the external force is acting opposite to the direction of motion of the body.

## 6.5 NUMERICAL PROBLEMS ON SECOND LAW OF MOTION

**Problem 6.1.** A force of unknown magnitude acts on a body of mass 150 kg and produce on acceleration of 3 $m/s^2$ in the direction of force. Find the above.

**Sol.** Given:
Mass of the body, $m = 150$ kg
Acceleration, $a = 3$ m/s$^2$

The force is given by equation (6.2),

Hence $F = m \times a = 150 \text{ (kg)} \times 3 \text{ (m/s}^2\text{)}$
$= 450 \text{ (kg - m/s}^2\text{)} =$ **450 N. Ans.** $(\because \text{kg - m/s}^2 = \text{N})$

**Problem 6.2.** A force of 100 N acts on a body having a mass of 4 kg for 10 seconds. If the initial velocity of the body is 5 m/s, determine:

(i) acceleration produced in the direction of force, and
(ii) distance moved by the body in 10 seconds.

**Sol.** Given:
Force, $F = 100$ N; Mass, $m = 4$ kg
Time, $t = 10$ second; Initial velocity $u = 5$ m/s

Let $a$ = Acceleration produced in the direction of force
$s$ = Distance travelled by body in 10 seconds.

(i) Using equation (6.2), we get

$$F = m \times a \quad \text{or} \quad 100 = 4 \times a$$

$$\therefore \quad a = \frac{100}{4} = \mathbf{25\ m/s^2.\ Ans.}$$

(ii) The distance moved is given by

$$s = ut + \frac{1}{2}at^2 = 5 \times 10 + \frac{1}{2} \times 25 \times 10^2$$
$$= 50 + 1250 = \mathbf{1300m.\ Ans.}$$

**Problem 6.3.** The weight of a body on earth is 980 N. If the acceleration due to gravity on earth = 9.80 m/s$^2$, what will be weight of the body on:

(i) the moon, where gravitational acceleration is 1.6 m/s$^2$, and
(ii) the sun, where gravitational acceleration is 270 m/s$^2$

**Sol.** Given:
Weight of body on earth, $W = 980$ N

Acceleration due to gravity on earth, $g = 9.80$ m/s$^2$

First calculate the mass of the body on earth. Using equation (6.1), we have

$$\text{Weight} = \text{mass} \times g \qquad \text{or} \qquad 980 = \text{mass} \times 9.80$$

$$\therefore \qquad \text{Mass} = \frac{980}{9.80} = 100 \text{ kg.}$$

The mass of the body will remain the same on moon as well as on sun.

(i) Weight of the body on moon, where $g = 1.6\ m/s^2$

Using equation (6.1),

$$\text{Weight} = \text{mass} \times g = 100 \times 1.6 = \mathbf{160\ N.\ Ans.}$$

(ii) Weight of the body on sun, where $g = 270\ m/s^2$

$$\text{Weight} = \text{mass} \times g = 100 \times 270 = \mathbf{27000\ N.\ Ans.}$$

**Problem 6.4.** A force of 200 N acts on a body having mass of 300 kg for 90 second. If the initial velocity of the body is 20 m/s, determine the final velocity of the body:

(i) when the force acts in the direction of motion and
(ii) when the force acts in the opposite direction of the motion.

**Sol.** Given:
Force, $F = 200$ N
Mass, $m = 300$ kg
Time, $t = 90$ seconds
Initial velocity $u = 20$ m/s.

Using equation (6.2), we have

$$\text{F} = m \times a \qquad \text{or} \qquad 200 = 300 \times a$$

$$\therefore \qquad a = \frac{200}{300} = \frac{2}{3} \text{ m/s}^2.$$

(i) Final velocity when the force acts in the direction of motion.

When the force acts in the direction of motion, the body will have acceleration. The final velocity ($v$) is given by

$$v = u + at = 20 + \frac{2}{3} \times 90 = 20 + 60 = \mathbf{80\ m/s.\ Ans.}$$

(ii) Final velocity when the force acts in the opposite direction of motion.

When the force acts in the opposite direction of motion, the body will have retardation. The final velocity is given by

$$v = u - at \qquad \text{(-ve sign shows retardation)}$$

$$= 20 - \frac{2}{3} \times 90 = 20 - 60 = -40 \text{ m/s}$$

(-ve sign shows that the body will be moving in the opposite direction).

**Problem 6.5.** A body of mass 15 kg falls on the ground from a height of 19.6 m. The body penetrates into the ground. Find the distance through which the body will penetrate into the ground, if the resistance by the ground to penetration is constant and equal to 4900 N. Take g = 9.8 m/s².

**Sol.** Given:

| | |
|---|---|
| Mass | $m = 15$ kg |
| Height of body from ground, | $h = 19.6$ m |
| Resistance to penetration, | $P = 4900$ N |

Let us first consider the motion of the body from a height of 19.6 m to the ground surface.

Initial velocity of the body, $u = 0$
Final velocity of the body, when it reaches the ground = $v$.

Using the equation,

$$v^2 - u^2 = 2gh \qquad \text{or} \qquad v^2 - 0 = 2 \times 9.8 \times 19.6$$

or $$v = \sqrt{2 \times 9.8 \times 19.6} = 19.6 \text{ m/s}.$$

When the body is penetrating into the ground, the resistance to penetration is acting in the upward direction (∴ Resistance always acts in the opposite direction of motion of the body). But the weight of the body is acting in the downward direction.

Weight of the body is given by equation (6.1),
Weight of the body = mass × $g$ = 15 × 9.80 N = 147 N
Upward resistance to penetration = 4900 N

∴ Net force acting in the upward direction,

$$F = 4900 - 147 = 4753 \text{ N}.$$

As the net force on the body is acting in the opposite direction to the motion of the body, this force will produce retardation.

Using equation (6.2), we have

$$\text{F} = m \times a \qquad \text{or} \qquad 4753 = 15 \times a$$

∴ $$a = \frac{4753}{15} = 316.866 \text{ m/s}^2.$$

Distance through which body will penetrate into the ground.

Consider the motion of the body from the ground to the point of penetration into the ground.

Let the distance of penetration = $s$

| | |
|---|---|
| Final velocity, | $v = 0$ |
| Initial velocity, | $u$= Velocity of the body on the ground = 19.6 m/s |
| Retardation, | $a = 316.866$ m/s² |

Using the relation,

$$v^2 - u^2 = -2a \times s \quad \text{(-ve sign is taken due to retardation)}$$

$$\therefore \quad 0 - 19.6^2 = -2 \times 316.866 \times s$$

$$\therefore \quad s = \frac{19.6 \times 19.6}{2 \times 316.866} = 0.606\text{m} = \textbf{60.6 cm. Ans.}$$

**Problem 6.6.** A man weighing 637 N dives into a swimming pool from a tower of height 19.6 m. He was found to go down in water by 2 m and then started rising. Find the average resistance of water. Neglect the resistance of air.

**Sol.** Given:
Weight of the man = 637 N
Height of tower, $h$ = 19.6 m

Distance travelled by man from the water surface into the water = 2 m.
First consider the motion of the man from the top of the tower to the water surface of the swimming pool.

Initial velocity of man, $u = 0$
Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$

Let the final velocity of the man, when he reaches the water surface = $v$.
Now using the relation,

$$v^2 - u^2 = 2gh \quad \text{or} \quad v^2 - 0^2 = 2 \times 9.80 \times 19.60$$

$$\therefore \quad v = \sqrt{2 \times 9.80 \times 19.60} = 19.6 \text{ m/s.}$$

Now consider the motion of the man from the water surface of the swimming pool upto the point, from where the man started rising.

Distance traversed, $s = 2$ m
Initial velocity of the man on the water surface
$u = 19.6$ m/s
Final velocity, $v = 0$.

As the velocity of the man becomes zero after travelling a distance 2 m inside the water, hence a force due to water resistance is acting in the opposite direction to the motion of the man. This force will produce retardation.

Let $a$ = Retardation due to water resistance

$\therefore$ Using the relation, $v^2 - u^2 = -2as$ (-ve sign is taken due to retardation)

or $$0^2 - 19.6^2 = -2 \times a \times 2$$

$$\therefore \quad s = \frac{19.6 \times 19.6}{2 \times 2} = 96.04 \text{ m/s}^2.$$

**Average resistance of water**

Let F* = Average resistance of water acting on the man in the upward direction.

Weight of man = 637 N acting in the downward direction

$\therefore$ Net force acting on the man in the upward direction

$$= F^* - \text{Weight of man } (F^* - 637) \text{ N.}$$

But the net force acting on the man must be equal to the product of mass of the man and retardation.

$$\therefore \quad (F^* - 637) = m \times a \qquad (\because F = m \times a) \qquad \ldots(i)$$

But mass of man, $m = \dfrac{\text{Weight of man}}{g}$ [using equation (6.1)]

$$= \frac{637}{9.8} = 65 \text{ kg} \quad \text{and} \quad a = 96.04 \text{ m/s}^2.$$

Substituting the values of $m$ and $f$ in equation (i), we get

$$(F^* - 637) = 65 \times 96.04$$

$$F^* = 65 \times 96.04 + 637 = \mathbf{6879.6 \text{ N. Ans.}}$$

**Problem 6.7.** A bullet of mass 81 gm and moving with a velocity of 300 m/s is fired into a log of wood and it penetrates to a depth of 10 cm. If the bullet moving with the same velocity, were fired into a similar piece of wood 5 cm thick, with what velocity would it emerge ? Find also the force of resistance, assuming it to be uniform.

**Sol.** Given:

Mass of bullet $m = 81 \text{ gm} = \dfrac{81}{1000} \text{ kg} = 0.081 \text{ kg}$

Initial velocity of bullet, $u = 300$ m/s

Distance travelled, $s = 10$ cm $= 0.10$ m

Final velocity, $v = 0$.

As the force of resistance is acting in the opposite direction of motion of bullet, hence force of resistance will produce retardation on the bullet.

Let $a$ = retardation

Now using the relation,

$$v^2 - u^2 = -2as \qquad \text{(-ve sign is taken due to retardation)}$$

or

$$0^2 - 300^2 = -2 \times a \times 0.1$$

$$\therefore \quad a = \frac{300 \times 300}{2 \times 0.1} = 450000 \text{ m/s}^2.$$

Let $F$ = Force of resistance offered by wood to the bullet.

Using equation (6.2), we get

$$F^* = m \times a = .081 \times 450000 = \mathbf{36450 \text{ N. Ans.}}$$

Velocity of the bullet with which the bullet will come out from a piece of wood of 5 cm thick.

Let $v$ = velocity with which the bullet emerges from the piece of wood of 5 cm thick.

Initial velocity, $u = 300$ m/s.

As the resistance offered by wood is uniform, hence retardation will be same as before.

$\therefore$ $a$ = 450000 m/s².
Distance travelled, $r$ = 5 cm = .05 m

Using the relation

$$v^2 - u^2 = -2as \quad \text{(-ve sign is taken due to retardation)}$$

$\therefore$ $0^2 - 300^2 = -2 \times 450000 \times 0.5$ or $v^2 = 300 \times 300 - 45000$
$= 90000 - 45000 = 45000$

$\therefore$ $v = \sqrt{45000}$ = **212.132 m/s. Ans.**

**Problem 6.8.** A car, moving on a straight level road, skidded for a total distance of 60 metres after the brakes were applied. Determine the speed of the car, just before the brakes were applied, if the co-efficient of friction between the car tyres and the road is 0.4. Take $g$ = 9.80 m/s².

**Sol.** Given:

Let $u$ = velocity of car just before applying the brakes
Final velocity of ear, $u$ = 0
Distance travelled, $s$ = 60 m

Co-efficient of friction between car tyres and road,
$\mu = 0.4$

Let $W$ = Weight of ear in Newton
$R$ = Normal reaction

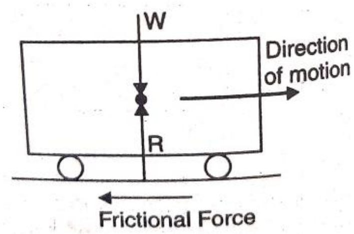


**Fig. 6.1**

Mass of car, $\frac{W}{g}$

Now frictional resistance

$= \mu R = \mu W \quad (\because R = W)$
$= 0.4$ W Newton
$F = 0.4$ W

As frictional force is acting in the opposite direction of motion, hence the frictional force will produce retardation.

Using equation (6.2).

Force = mass × a

$\therefore$ $0.4\text{ W} = \frac{W}{g} \times a$ or $a = \frac{0.4\text{ W} \times g}{W} = 0.4 \times 9.80 = 3.92$ m/s²

$\therefore$ Retardation = 3.92 m/s²

Now using the relation,

$$v^2 - u^2 = -2as$$

$\therefore$ $0^2 - u^2 = -2 \times 3.92 \times 60$

$\therefore$ $u = \sqrt{2 \times 3.92 \times 60} = \sqrt{4704} = 21.688$ m/s

$$= \frac{21688}{100} \times 60 \times 60 \text{ km/hour} = \textbf{78.077 km/hour Ans.}$$

**Problem 6.9.** The tractive force, exerted by a railway car weighing 50 kN, is 2000 N. If the frictional resistance is 5 N per kN of the railway car's weight, determine the acceleration when the railway car is moving on a level track.

**Sol.** Given :

Tractive force exerted by railway car,

$$F_1 = 2000\text{N}$$

Weight of car, $W = 50 \text{ kN} = 50 \times 1000\text{N}$

$\therefore$ Mass of car, $m = \frac{W}{g} = \frac{50 \times 1000}{9.81}$ kg

Frictional resistance, $F_2$ = 5 N per kN of car's weight

$= 5 \text{ N} \times \text{weight of car in kN} = 5 \times 50 \text{ N} = 250 \text{ N}.$

The tractive force is acting in the direction of motion, while frictional resistance is acting in opposite direction of motion.

$\therefore$ Net force in the direction of motion,

$$F = F_1 - F_2 = 2000 - 250 = 1750 \text{ N}.$$

As the net force is acting in the direction of motion, it will produce acceleration.

Let $a$ = acceleration produced

Using equation (6.2), we have

$$F = m \times a$$

or $$1750 = \frac{50 \times 1000}{9.81} \times a \qquad \left(\because m = \frac{50 \times 1000}{9.81}\right)$$

$\therefore$ $$a = \frac{1750 \times 9.81}{50 \times 1000} = \textbf{0.343 m/s}^2\textbf{. Ans.}$$

**6.5.1. Motion on an Inclined Smooth Surface.** Fig. 6.2 shows a body of weight W, sliding down on a smooth inclined plane.

Let $\theta$ = Angle made by inclined plane with horizontal

W = Weight of the body

$a$ = Acceleration of the body

$m$ = Mass of the body

$= \frac{W}{g}$

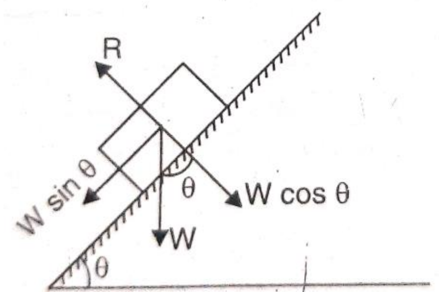


**Fig. 6.2**

As the surface of the plane is smooth, hence the frictional force will be zero. Hence the forces acting on the body are its own weight Wand reaction R of the plane. The resolved part of W perpendicular to the plane is W cos θ, which is balanced by R, while the resolved part parallel to the plane is W sin θ, which produces acceleration down the plane. This force is responsible for the movement of the body down the plane.

Net force acting on the body down the plane is

$$\therefore \qquad F = W \sin \theta$$

Now using the equation (6.2), we have

$$F = m \times a$$

Substituting the values of F and m in the above equation, we get

$$W \sin \theta = \frac{W}{g} \times a$$

$$\therefore \qquad a = g \sin \theta \qquad \ldots(6.3)$$

If the body is moving up the plane, the corresponding acceleration will be – g sin θ.

**6.5.2. Motion on an Inclined Rough Surface.** Fig. 6.3 shows a body of weight W, sliding down the rough inclined surface.

Let $a$ = Acceleration of the body

$m$ = Mass of the body = $\frac{W}{g}$

θ = Inclination of the plane with horizontal

μ = Co-efficient of friction.

$F_1$ = Force of friction.

As the body is moving down the plane, the force of friction will be acting up the plane as shown in Fig. 6.3.

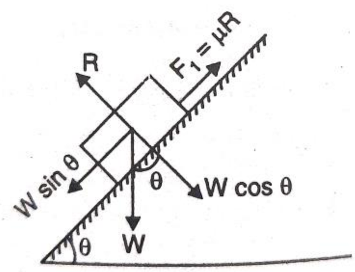

**Fig. 6.3**

Now force of friction,

$$F_1 = \mu R$$

$$= \mu \times W \cos \theta$$

Force acting down the plane,

$$F_2 = W \sin \theta$$

∴ Net force acing on the body down the plane,

$$F = F_2 - F_1 = W \sin \theta - \mu W \cos \theta$$

Now using the equation (6.2),

$$F = m \times a$$

or $\quad (W \sin\theta - \mu W \cos\theta) = \frac{W}{g} \times a \qquad (\because m = \frac{W}{g})$

or $\quad W = (\sin\theta - \mu\cos\theta) = \frac{W}{g} \times a$ or $a = W(\sin\theta - \mu\cos\theta) \times \frac{g}{W}$

$\therefore \quad a = g(\sin\theta - \mu\cos\theta) \qquad ...(6.4)$

**Problem 6.10.** A body of weight 200 N is initially stationary on a 45° inclined plane. What distance along the inclined plane must be body slide, before it reaches a speed of 2 m/s. The co-efficient of friction between the body and the plane = 0.1.

**Sol.** Given :

| | |
|---|---|
| Weight of body | W = 200 N |
| ∴ Mass of body, | $m = \frac{W}{g} = \frac{200}{9.81}$ kg |
| Angle of plane, | θ = 45° |
| Initial velocity, | $u = 0$ |
| Final velocity, | $v = 2$ m/s |
| Co-efficient of friction, | μ = 0.1. |

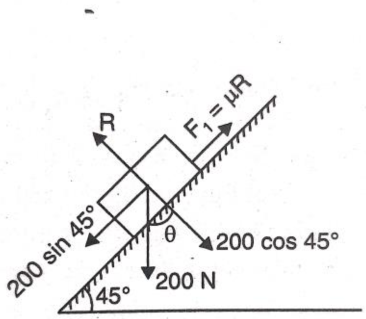


**Fig. 6.4**

The acceleration of the body is given by equation (6.4) as

$$a = g[\sin\theta - \mu\cos\theta]$$
$$= 9.81\,[\sin 45^\circ - 0.1 \cos 45^\circ]$$
$$= 9.81\,[0.707 - 0.1 \times .707]$$
$$= 6.242 \text{ m/s}^2.$$

Now using the relation

$v^2 - u^2 = 2as \qquad$ or $\qquad 2^2 - 0^2 = 2 \times 6.242 \times s$

$\therefore \qquad s = \frac{2 \times 2}{2 \times 6.242} = 0.32$ m = **32 cm. Ans.**

## 6.6 LIFE MOTION

Fig. 6.5 shows a lift (elevator or cage) carrying some weight and moving with a uniform acceleration.

Let W = Weight carried by the lift

$m$ = Mass carried by lift = $\frac{W}{g}$

$a$ = Uniform acceleration of the lift

T = Tension in cable supporting the lift. This is also called the *reaction of the lift.

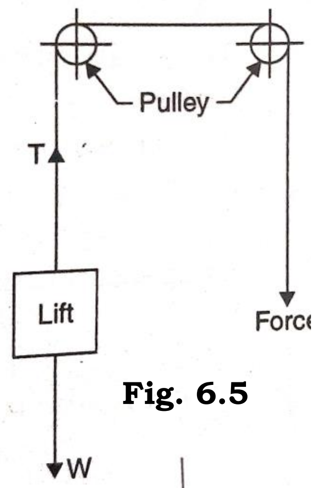


**Fig. 6.5**

The lift may be moving upwards or moving downwards.

**1st Case.** Let the lift is moving upwards as shown in Fig. 6.6. The weight carried by lift is acting downwards while the tension in the cable is acting upwards. As the lift is moving up, the net force which is equal to (T – W) is acting upwards.

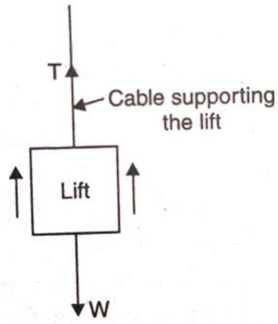


**Fig. 6.6** Lift is moving upwards

∴ Net force in upward direction = T - W.

This net force produces an acceleration '$a$'

Hence using,

Net force = mass × acceleration

or $$(T - W) = \left(\frac{W}{g}\right) \times a \qquad \left(\because \text{Mass} = \frac{W}{g}\right)$$

or $$T = W + \frac{W}{g} \times a = W\left(1 + \frac{a}{g}\right) \qquad \ldots(6.5)$$

**2nd Case.** As the lift is moving downwards as shown in Fig. 6.7, the net force is acting downwards. Hence in this case W is more than T (tension in string).

∴ Net force in downward direction = (W - T).

This net force produces an acceleration '$a$'.

Hence using, Net force = mass x acceleration

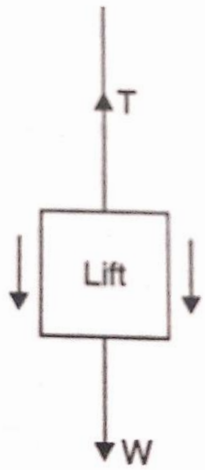


**Fig. 6.7** Lift is moving downwards

or $$(W - T) = \frac{W}{g} \times a$$

or $$T = W - \frac{W}{g} \times a$$

$$= W\left(1 + \frac{a}{g}\right) \qquad \ldots(6.6)$$

**6.6.1. Problem Based on Lift Motion**

**Problem 6.11.** A lift carries a weight of 100 N and is moving with a uniform acceleration of 2.45 m/s$^2$. Determine the tension in the cables supporting the lift, when:

(i) lift is moving upwards, and
(ii) lift is moving downwards. Take g = 9.80 m/s$^2$.

**Sol.** Given :

Weight carried by lift, W = 100 N.

Uniform acceleration, $a$ = 2.45 m/s$^2$.

(i) Lift is moving upwards

Let T = Tension in the cables supporting the lift.

Using equation (6.5), $T = W\left(1 + \frac{a}{g}\right) = 100\left(1 + \frac{245}{9.80}\right)$

$= 100(1.25) =$ **125 N. Ans.**

(ii) Lift if moving downwards

Using equation (6.6), $T = W\left(1 + \frac{a}{g}\right) = 100\left(1 + \frac{245}{9.80}\right) = 100(1 - 0.25)$

$= 100 \times .75 =$ **75 N Ans.**

**Problem 6.12.** A life has an upward acceleration of 1.225 m/s$^2$. What pressure will a man weighing 500 N exert on the floor of the lift? What pressure would he exert if the lift had an acceleration of 1.225 m/s$^2$ downwards ? What upward acceleration would cause his weight to exert to pressure of 600 N on the floor? Take G = 9.8 m/s$^2$.

**Sol.** Given:

Upward acceleration, $a$ = 1.225 m/s$^2$

Weight of man, W = 500 N.

**1st Case.** The lift is moving up with an acceleration of 1.225 m/s$^2$. The pressure exerted by a man on the floor of the lift is equal to the reaction of the lift and it is the same as the tension in the cables supporting the lift.

Let T = Tension in the cables supporting the lift or the reaction of the lift or the pressure exerted by the man on the floor of the lift.

Using equation (6.5) for the lift moving upwards, we have

$T = W\left(1 + \frac{a}{g}\right) = 500\left(1 + \frac{1.225}{9.80}\right) = 500(1 + .125) =$ **562.5 N. Ans.**

**2nd Case.** The lift is moving downwards with an acceleration of 1.225 m/s$^2$. The pressure exerted by the man on the floor of the lift is equal to the reaction of the lift and it is the same as the tension in the cables supporting the lift.

$T = W\left(1 + \frac{a}{g}\right) = 500\left(1 + \frac{1.225}{9.80}\right) = 500(1 - .125) =$ **437.5 N. Ans.**

**3rd Case.** The lift is moving upwards with an unknown acceleration.

Let $a$ = Acceleration upwards

T = Pressure exerted by man on floor of lift = 600 N

W = Weight of man = 500 N.

Using equation (6.5), we get

$$T = W\left(1 + \frac{a}{g}\right) \quad \text{or} \quad 600 = 500\left(1 + \frac{a}{9.80}\right)$$

or $$\frac{600}{500} = 1 + \frac{a}{9.80} \quad \text{or} \quad 1.2 = 1 + \frac{a}{9.80}$$

or $a = (1.2 - 1.0) \times 9.80 = 0.2 \times 9.80 =$ **1.96 m/s². Ans.**

**Problem 6.13.** An elevator weighs 2500 N and is moving vertically downwards with a constant acceleration. Write the equation for the elevator cable tension. Starting from rest it travels a distance of 35 metres during an interval of 10 seconds. Find the cable tension during this time, Neglect all other resistances to motion. What are the limits of cable tension ?

**Sol.** Given :

Weight of elevator, W = 2500 N

Initial velocity, $u = 0$

Distance travelled, s = 35 m

Time, $t = 10$ sec.

Let T = Tension in the cable supporting an elevator in N.

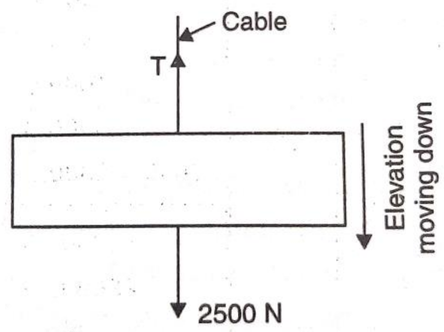


**Fig. 6.8**

**1st Part.** The equation for the elevator cable tension is obtained as given below : (see Fig. 6.8). The elevator is moving down.

∴ Net acceleration force in the downward direction

= (W – T) = (2500 – T) N.

The net accelerating force produces an acceleration '$a$' in the downward direction.

Hence using the relation

Net force = mass × acceleration

$$(2500 - T) = \frac{2500}{9.81} \times a$$

or $$T = 2500 - \frac{2500}{9.81} \times a$$

∴ $$T = 2500\left(1 + \frac{a}{9.80}\right)\text{N}$$

Hence the above equation (i) represents the equation for the elevator cable tension when the elevator is moving downwards.

**2nd Part.** Limits of cable tension are obtained from equation (i) as given below:

(i) When a = 0, and this value is substituted in equation (i), the value of T is obtained as

$$T = 2500\left(1 + \frac{a}{9.81}\right) = 2500 \text{ N.}$$

(ii) When a = 9.81 m/s², and this value is substituted in equation (i), the value of T is obtained as

$$T = 2500\left(1 + \frac{9.81}{9.81}\right) = 2500\ (1 - 1) = 0$$

Limits of cable tension (T) are :

At $a$ = 0, T = 2500 N
At $a$ = 9.81, T = **0. Ans.**

**3rd Part.** S = 35 m, $u$ = 0 and $t$ = 10 seconds.

Using the relation, $s = ut + \frac{1}{2}ut^2$

or $$35 = 0 \times 10 + \frac{1}{2} \times a \times 10^2 = 0 + 50\ a$$

$$\therefore \quad a = \frac{35}{50} = 0.7 \text{ m/s}^2$$

Substituting this value off in equation (1), we get

$$T = 2500\left(1 + \frac{0.7}{9.81}\right) = \mathbf{2321.61\ N.\ Ans.}$$

**Problem 6.14.** A cage, carrying 10 men each weighing 500 N, starts moving downwards from rest in a mine vertical shaft. The cage attains a speed of 12 metresls in 20 metres. Find the pressure exerted by each man on the floor of the cage. Take g = 9.80 m/s²

**Sol.** Given :

| | |
|---|---|
| Weight of one man, | = 500 N |
| Total weight of 10 men on the cage, | W = 500 × 10 = 5000 N |
| Initial velocity of cage, | $u$ = 0 |
| Final velocity of cage, | $v$ = 12 m/s |
| Distance travelled, | s = 20 m |
| Acceleration due to gravity, | g = 9.80 m/s² |

The pressure exerted by the men on the floor of the cage will be same as the tension produced in the cables supporting the cage.

Let T = Total tension produced by 10 men in the cables.
$O$ = Acceleration of the cage.

The acceleration will be obtained by using the relation.

$$v^2 - u^2 = 2as \qquad \text{or} \qquad 12^2 - 0 = 2a \times 20$$

$$\therefore \qquad a = \frac{12 \times 12}{2 \times 20} = 3.6 \text{ m/s}^2$$

The cage is moving downwards hence using equation (6.6),

$$T = W\left(1 + \frac{a}{g}\right) \qquad \text{or} \qquad T = 5000\left(1 - \frac{3.6}{9.8}\right)$$

$$= 5000\,(1 - 0.3673) = 3163.26 \text{ N.}$$

∴ Tension produced by one man in the cable,

$$\frac{T}{10} = \frac{3163.26}{10} = 316.326 \text{ N.}$$

But tension produced by each man in the cable is the same as the pressure exerted by each man on the floor of the cage.

∴ Pressure exerted by each man on the floor = **316.326 N. Ans.**

**Problem 6.15.** As elevator weighing 5000 N is ascending with an acceleration of 3 m/s$^2$. During this ascent its operator whose weight is 700 N is standing on the scales placed on the floor. What is the scale reading? What will be the total tension in the cables of the elevator during the motion?

**Sol.** Given:

Weight of the elevator, $W_1$ = 5000 N

Acceleration of elevator, $a$ = 3 m/s$^2$

Weight of the operator, $W^2$ = 700 N

When the operator is standing on the scale placed on the floor of the elevator, the reading the scale will equal to the reaction (R) offered by the floor on the operator.

Hence let R = Reaction offered by the floor on operator. This is also equal to the reading of scale.

T = Total in the cables of elevator.

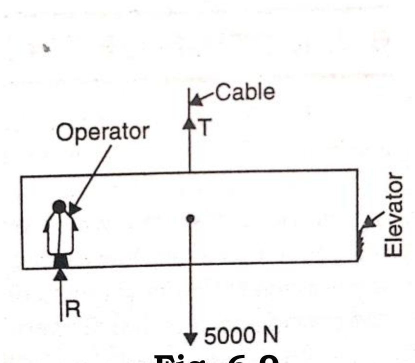


**Fig. 6.9**

Consider the motion of operator. The operator is moving upwards along with the elevator with an acceleration, $a$ = 3 m/s$^2$. The net force on the operator is acting upwards.

∴ Net upward force on operator

= Reaction offered by floor on operator

- Weight for operator

= (R – 700)

Mass of operator $= \frac{\text{Weight of operator}}{g} = \frac{700}{g}$

But, net force = × acceleration$\frac{3163.26}{10}$

$\therefore$ $(R - 700) = \frac{700}{9.8} \times 3$

$\therefore$ $R = 700 + \frac{700}{9.8} \times 3 = 700 + 214.28$

$= \mathbf{914.28\ N.\ Ans.}$

Total tension in the cables of elevator.

Let T = Total tension in the cables of elevator

W = Total weight (i.e., weight of elevator + weight of operator)

As the elevator with the operator is moving upwards with an acceleration $f = 3\ m/s^2$, the net force will be acting on the elevator and operator in the upward direction.

$\therefore$ Net upwards force on elevator and operator

= Total tension in the cables – Total weight of elevator and operator

= (T – 5700)

Mass of elevator and operator

$= \frac{\text{Total weight}}{g} = \frac{5700}{9.80}$

But net force = mass × acceleration

$\therefore$ $(T - 500) = \frac{5700}{9.8} \times 3$

$\therefore$ $T = 5700 + \frac{5700}{9.8} \times 3 = 5700 + 1745 =$ **7445 N. Ans.**

## 6.7 MOTION OF TWO BODIES TIES BY A STRING

Fig. 6.10 shows a light and inextensible string passing over a smooth and weight- less pulley. Two bodies of weights $W_1$ and $W_2$ are attached to the two ends of the string. Let $W_1$ be greater than $W_2$. As $W_1 > W_2$ the weight $W_1$ will move downwards, whereas the smaller weight ($W_2$) will move upwards. For an inextensible string, the up- ward acceleration of the weight W, will be equal to the downward acceleration of the weight $W_1$

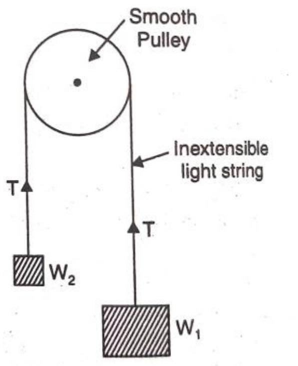


**Fig. 6.10**

As the string is light and inextensible and passing over a smooth pulley, the tension* of the string will be the same on both sides of the pulley.

Let T = Tension in both strings,
$a$ = Acceleration of the bodies.

Consider the motion of weight $W_1$. The weight $W_1$ is moving downwards with an acceleration $a$. The forces acting on $W_1$ are (i) its weight $W_1$ acting downwards and (ii) tension T acting upwards. As the weight $W_1$ is moving downwards, hence net force on the weight $W_1$ is acting downwards.

$\therefore$ Net downward force = $(W_1 = T) = (m_1 \times g \times T)$

But net force = mass × acceleration

$\therefore$ Weight = mass × $g$

$\therefore$ $W_1 = m_1 \times g$

$$(W_1 - T) = \frac{W_1}{g} \times a \qquad (\because \text{Mass} = \frac{\text{Weight}}{g} = \frac{W_1}{g}) \quad \ldots(i)$$

Now consider the motion of weight W2. The forces acting on $W_2$ are: (i) its weight $W_2$ acting downwards and (ii) tension T acting upwards. But the weight $W_2$ is moving upwards, hence net force on weight $W_2$ is acting upwards.

Net upward force = $(T - W_2)$
But net upward force = mass × acceleration

or $$(T - W_2) = \frac{W_2}{g} \times a \qquad (\because \text{Mass} = \frac{W_1}{g}) \quad \ldots(ii)$$

Adding equations (i) and (ii), we get

$$(W_1 - W_2) = \frac{a}{g} \times (W_1 - W_2)$$

or $$a = \frac{g(W_1 - W_2)}{(W_1 + W_2)} \quad \ldots(6.7)$$

Equation (6.7) is used for finding the acceleration. If the value of this acceleration is substituted either in equation (i) or in equation (ii) the value of tension (T) is obtained.

Hence substituting the value of 'a' in equation (ii), we get

$$(W_1 - T) = \frac{W_1}{g} \times g\frac{(W_1 - W_2)}{(W_1 + W_2)} = \frac{W_1(W_1 - W_2)}{(W_1 + W_2)} \text{ (Cancelling g)}$$

or $$T = W_1 - \frac{W_1(W_1 - W_2)}{(W_1 + W_2)} = W_1\left[1 - \frac{(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$= W_1\left[\frac{W_1 + W_2 - W_1 + W_2)}{(W_1 + W_2)}\right] = \frac{2W_1W_2}{(W_1 + W_2)} \qquad \ldots(6.8)$$

**Problem 6.16.** Two bodies of weight 50 N and 30 N are connected to the two ends of a light inextensible string. The string is passing over a smooth pulley. Determine:

(i) The acceleration of the system, and
(ii) Tension in the string. Take g = 9.80 m/s2.

**Sol.** Given :

Bigger weight, $W_1 = 50$ N

Smaller weight, $W_2 = 30$ N

As $W_1 > W_2$ hence weight 50 N is moving downwards whereas weights 30 N is moving upwards.

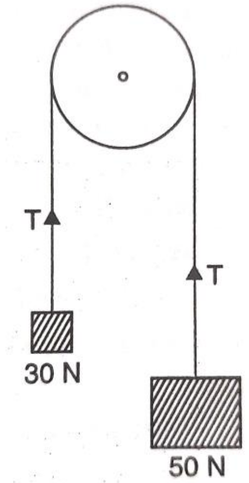


**Fig. 6.11**

Let $a$ = Acceleration of the system, and
T = Tension in the sting.

(i) Using the equation (6.7) for acceleration,

$$a = \frac{g(W_1 - W_2)}{(W_1 + W_2)} = \frac{9.80(50-30)}{(50+30)}$$

$$= \frac{9.8 \times 20}{80} = \mathbf{2.45\ m/s^2\ Ans.}$$

(ii) Using equation (6.8) for tension in the string.

$$T = \frac{2W_1W_2}{(W_1 + W_2)} = \frac{2 \times 50 \times 30}{(50+30)} = \frac{2 \times 50 \times 30}{80}$$

$$= \mathbf{37.5\ N.\ Ans.}$$

**Problem 6.17**. Two bodies of different weights are connected to the two ends of a light inextensible string, which passes over a smooth pulley. If the acceleration of the system is 3 m/s$^2$ and bigger weight is 60 N, determine:

(i) The smaller weight, and
(ii) Tension in the string. Take g = 9.80 m/s$^2$.

**Sol.** Given :

Acceleration, $a = 3$ m/s$^2$
Bigger weight, $W_1 = 60$ N

Let $W_2$ = Smaller weight, and
T = Tension in the string.

(i) Using equation (6.7), we get

$$a = \frac{g(W_1 - W_2)}{(W_1 + W_2)} \quad \text{or} \quad 3 = \frac{9.80(60 - W_2)}{(60 + W_2)}$$

or $\quad 3\ (60 + W_2) = 9.80\ (60 - W_2)$

or $\quad 180 + 3W_2 = 9.80 \times 60 - 9.80W_2$

or $\quad 3W_2 + 9.80W_2 = 9.80 \times 60 - 180 \quad \text{or} \quad 12.80W_2 = 408$

$\therefore \quad W_2 = \frac{408}{12.80} =$ **31.875 N. Ans.**

(ii) Tension in the string is obtained from equation (6.8),

$$T = \frac{2W_1W_2}{(W_1 + W_2)} = \frac{2 \times 60 \times 31.875}{(60+31.875)} = \frac{2 \times 60 \times 31.875}{91.875} = \mathbf{41.632\ N.\ Ans.}$$

**Problem 6.18.** A pulley whose axis passes through the centre O, carries load as shown in Fig. 6.12. Neglecting the inertia of the pulley and assuming that the card is inextensible, determine the acceleration of the block A. How much weight should be added to or taken away from the block A, if the acceleration of the block A is required to be g/3.0 downwards?

**Sol.** Given:

Bigger load, $W_1 = 700$ N

Smaller load, $W_2 = 700$ N

Let $a$ = Acceleration of block. A or the acceleration of the system.

Using equation (6.17),

$$a = \frac{g(W_1 - W_2)}{(W_1 + W_2)}$$

$$= \frac{g(700 - 500)}{(700 + 500)} = g\frac{200}{1200} = \frac{\mathbf{g}}{\mathbf{6}}\mathbf{.\ Ans.}$$

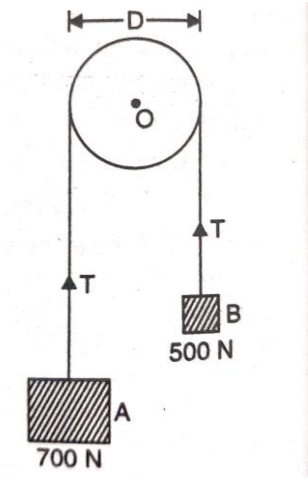


**Fig. 6.12**

How much weight should be added to or taken away from the block A (i.e., from bigger load 700 N) when acceleration of bigger load is g/3.0 downwards.

Let $W_1^*$ = Total weight of block A when acceleration is g/3.0

$$a = \frac{g}{3} \quad W_2 = 500 \text{ N}$$

Using equation (6.7), $a = \frac{g(W_1^* - W_2)}{(W_1^* + W_2)}$ $\quad (\therefore \text{Here } W_1 - W_1^*)$

or $$\frac{g}{3} = \frac{g(W_1{}^* - 500)}{(W_1{}^* + 500)}$$

or $$\frac{1}{3} = \frac{(W_1{}^* - 500)}{(W_1{}^* + 500)} \qquad \text{(cancelling } g \text{ to both sides)}$$

or $$3(W_1{}^* - 500) = W_1{}^* + 500 \text{ or } \quad 3W_1{}^* - 1500 = W_1{}^* + 500$$

or $$2W_1{}^* = 2000$$

$$\therefore \quad W_1{}^* = \frac{2000}{2} = 1000 \text{ N.}$$

As $W_1{}^*$ is more than W. Hence the weight must be added to the block A.

$\therefore$ Weight added $= W_1{}^* - W_1 = 1000 - 700 =$ **300 N. Ans.**

**6.7.1. Motion of Two Bodies Connected by a String when One Body is lying on a Horizontal Surface and other is Hanging Free**

**1. The horizontal surface is smooth and the string is passing over a smooth pulley.** Fig. 6.13 shows the two weights $W_1$ and $W_2$ connected by a light inextensible string, passing over a smooth pulley. The weight $W_2$ is placed on a smooth horizontal surface, whereas the weight W, is hanging free.

The weight $W_1$ is moving downwards, whereas the weight $W_2$ is moving on smooth horizontal surface. The velocity and acceleration of $W_1$ will be same as that of $W_2$.

As the string is light and inextensible and passing over a smooth pulley, the tensions of the string will be same on both sides of the pulley.

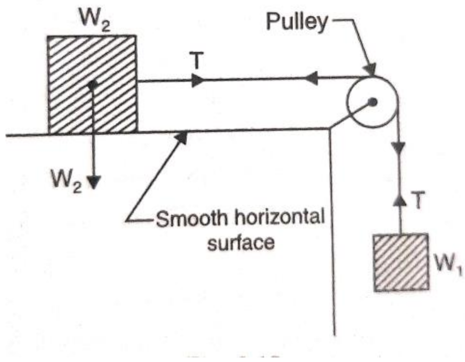


**Fig. 6.13**

Let T = Tension in the string

$a$ = Acceleration of the weight $W_1$ and also of $W_2$

(i) Consider the motion of the hanging weight $W_2$.

The weight $W_1$ is moving downwards with an acceleration a. The forces acting on $W_1$ are: (i) its weight $W_1$ acting downwards, and (ii) tension T acting upwards.

$\therefore$ Net downward force $= (W_1 - T)$

Using, net force = mass × acceleration

$$(W_1 - T) = \frac{W_1}{g} \times a \qquad (\because \text{Mass} = \frac{\text{Weight}}{g}) \ldots\text{(i)}$$

(ii) Consider the motion of weight $W_2$

The weight $W_2$ is moving on the horizontal surface with an acceleration of '$a$'. As the weight $W_2$ is moving in the horizontal the only force causing the motion is T. The weight $W_2$ is acting downwards and hence the component of this weight in horizontal direction is $W_2 \cos 90°$, which is zero.

Using, force = mass × acceleration

or $$T = \frac{W_1}{g} \times a \qquad (\therefore \text{Mass} = \frac{\text{Weight}}{g}) \quad \ldots(i)$$

Adding equations (i) and (ii),

$$W_1 = \frac{W_1}{g} \times a + \frac{W_2}{g} \times a = \frac{a}{g} \times [W_1 + W_2]$$

$$\therefore \quad a = \frac{g \times W_2}{(W_1 + W_2)} \quad \ldots(6.9)$$

Equation (6.9) gives the acceleration of the system.

To find the tension (T) in the sting, subsuming the value '$a$' in equation (ii).

$$\therefore \quad T = \frac{W_1}{g} \times \frac{g \times W_2}{(W_1 + W_2)} = \frac{W_1 W_2}{(W_1 + W_2)} \quad \ldots(6.10)$$

Equation (6.10) gives the tension in the string.

**2. The horizontal surface is rough and string is passing over a smooth pulley**. Fig 6.14 shows the two weights $W_1$ and $W_2$ connected by a light inextensible string, passing over a smooth pulley. The weight $W_1$ is hanging free, whereas the weight $W_2$ is placed on a rough horizontal surface. Hence in this case force of friction will be acting on the weight $W_2$ in the opposite direction of the motion of weight $W_2$ as shown in Fig. 6.14.

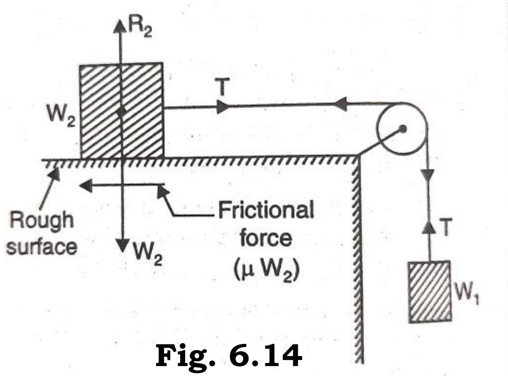


**Fig. 6.14**

Let $\mu$ = Co-efficient of friction between weight W, and horizontal surface.

$a$ = Acceleration of the system

T = Tension in the string

$R_2$ = Normal reaction at the horizontal rough surface = $W_2$

Force of friction = $\mu R_2$

$$= \mu W_2 \qquad (\therefore R_2 = W_2) \quad \ldots(i)$$

(i) Consider the motion the hanging weight $W_1$)

The weight $W_1$ is moving downwards with an acceleration '$a$'. The net downward force acting on weight $W_1 = (W_1 - T)$

Using net force = mass × acceleration

$$W_1 - T = \frac{W_1}{g} \times a \qquad (\because \text{Mass} = \frac{\text{Weight}}{g}) \qquad \ldots\text{(ii)}$$

(ii) Consider the motion of the weight $W_2$

The weight $W_2$ is moving on the rough horizontal surface towards right with an acceleration '$a$'. The forces acting in the horizontal direction are : (i) tension (T) towards right, and (ii) force of friction = $\mu R_2 = \mu W_2$ towards left.

$\therefore$ Net horizontal force towards right = $T - \mu W_2$

Using net force = mass × acceleration

Or
$$T - \mu W_2 = \frac{W_2}{g} \times a \qquad \ldots\text{(iii)}$$

Adding Equations (ii) and (iii)

$$W_1 - \mu W_2 = \frac{a}{g}(W_1 + W_2)$$

$$\therefore \qquad a = g\,\frac{(W_1 - \mu W_2)}{(W_1 + W_2)}\ \text{m/s}^2 \qquad \ldots(6.11)$$

Equation (6.11) is used to find the acceleration

To find the tension T, substitute the value of '$a$' in equation (ii)

$$\therefore \qquad W_1 - T = \frac{W_1}{g} \times a \quad \frac{g(W_1 - \mu W_2)}{(W_1 + W_2)} = \frac{W_1(W_1 - \mu W_2)}{(W_1 + W_2)}$$

$$T - W_1 = \frac{W_1(W_1 - \mu W_2)}{(W_1 + W_2)} = W_1\left[1 - \frac{(W_2 - \mu W_2)}{(W_1 + W_2)}\right]$$

$$= W_1\left[\frac{W_1 + W_2 - (W_1 - \mu\, W_2)}{W_1 + W_2}\right]$$

$$= \frac{W_1}{(W_1 + W_2)}(W_1 + W_2 - W_1 + \mu\, W_2)$$

$$= \frac{W_1(W_1 + \mu W_2)}{(W_1 + W_2)} = \frac{W_1 W_1(1 + \mu)}{(W_1 + W_2)} \qquad \ldots(6.12)$$

**Problem 6.19.** Two bodies of weight 20 N and 10 N are connected to the two ends of a light inextensible string, passing over a smooth pulley.

The weight of 20 N is placed on a smooth horizontal surface while the weight of 10 N is hanging free in air. Find :

(i) the acceleration of the system, and
(ii) the tension in the string. Take g = 9.81 m/s$^2$.

**Sol.** Given :

Weight placed on horizontal surface, $W_2$ = 20 N

Weight hanging free in air, $W_1$ = 10 N

Acceleration due to gravity, $g$ = 9.81 m/s$^2$

Let a = Acceleration of the system, and
T = Tension in the string.

The horizontal surface is smooth. Hence the acceleration and tension are obtained by using equation (6.9) and (6.10).

Using equation (6.9) for acceleration, we have

$$a = \frac{g\ W_1}{(W_1 + W_2)} = \frac{9.81 \times 10}{(10+20)} = \mathbf{3.27\ m/s^2.\ Ans.}$$

$$T = \frac{W_1 W_2}{(W_1 + W_2)} = \frac{10 \times 20}{(10+20)} = \frac{200}{30} = \mathbf{6.67\ N\ Ans.}$$

**Problem 6.20.** If in the problem 6.19, the horizontal surface is a rough one, having co-efficient of friction between the weight 20 N and the plane surface equal to 0.3 determine:

(i) the acceleration of the system, and
(ii) the tension in the string.

**Sol.** Given : (From Problem 6.19)

$W_1$ = 10 N, $W_1$ = 20 N, g = 9.81 m/s$^2$.

Co-efficient of friction between weight 20 N and horizontal surface,

$\mu$ = 0.3

Let $a$ = Acceleration of the system, and
T = Tension in the string.

For a rough horizontal surface, the acceleration and tension are obtained by using equations (6.11) and (6.12).

Using equation (6.11) for acceleration, we have

$$a = \frac{g\ (W_1 - \mu W_2)}{(W_1 + W_2)} = \frac{9.81(10-0.3 \times 20)}{(10+20)} = \frac{9.81 \times 4}{30} = \mathbf{1.308\ m/s^2.\ Ans.}$$

Using equation (6.12) for tension, we have

$$T = \frac{W_1 W_2 (1+\mu)}{(W_1 + W_2)} = \frac{10 \times 20\ (1+0.3)}{(10+20)} = \frac{200 \times 13}{30} = \mathbf{8.66\ N.\ Ans.}$$

**Problem 6.21.** Two blocks shown in Fig. 6.15 have weights A = 20 N and B = 10 N and co-efficient of friction between the block A and the horizontal plane is $\mu$ = 0.25. If the system is released from the rest and the block B falls through a vertical distance of 2 m, what is the velocity attained by block B. Neglect the friction in the pulley and the extension of the string.

**Sol.** Given :

Weight of block A, $W_2 = 20$ N

Weight of block B, $W_1 = 10$ N

Co-efficient of friction, $\mu = 0.25$

Distance moved by block B, $s = 2$ m

Initial velocity of block B, $u = 0$.

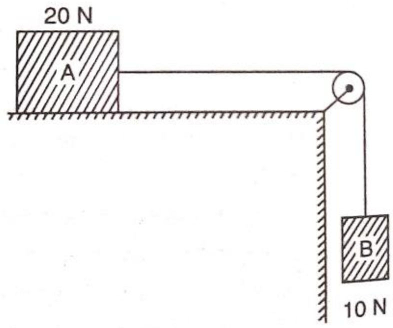


**Fig. 6.15**

Let $v$ = Final velocity of block B

$a$ = Acceleration of the system or acceleration of block B.

Using equation (6.11) for acceleration, we have

$$a = \frac{g\ (W_1 - \mu W_2)}{(W_1 + W_2)} = \frac{9.81(10-0.25 \times 20)}{(10+20)} = \frac{9.81 \times 5}{30} = \mathbf{1.635\ m/s^2.\ Ans.}$$

Now using the relation,

$$v^2 = u^2 + 2as = 0 + 2 \times 1.635 \times 2 = 6.54$$

$$\therefore \quad v = \sqrt{6.54} = \mathbf{2.557\ m/s\ Ans.}$$

**Problem 6.22**. Two bodies of weight 10 N and 1.5 N are connected to the two ends of a light inextensible string, passing over a smooth pulley. The weight 10 N is placed on a rough horizontal surface while the weight of 1.5 N is hanging vertically in air. Initially the friction between the weight 10 N and the table is just sufficient to prevent motion. If an additional weight of 0.5 N is added to the weight 1.5 N, determine :

(i) the acceleration of the two weights, and

(ii) tension in the string after adding additional weight of 0.5 N to the weight 1.5 N. take g = 9.80 m/s²

**Sol.** Given :

Weight placed on rough horizontal surface, $W_2 = 10$ N

Weight hanging free in air, $W_1^* = 1.5$ N

Additional weight added to $W_1 = 0.5$ N

∴ Total weight hanging in air in second case.

$$W_1 = 1.5 + 0.5 = 2.0 \text{ N}$$

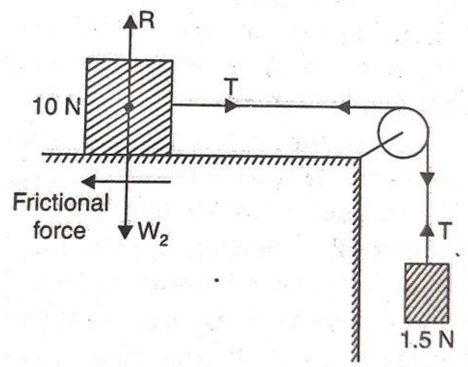


**Fig. 6.16**

Let T = Tension in string when hanging weight is 1.5 N

R = Normal reaction between the weight 10 N and the table surface

$T_1$ = Tension in string when hanging weight is 2.0 N

$a$ = Acceleration of the system when hanging weight is 2.0 N

Initially, the friction between the weight 10 N and the table is just sufficient to prevent motion.

∴ Max. Frictional Force, F = T = 1.5 N.

But frictional force, F = μR

or $$1.5 = \mu \times 10 \qquad (\because R = W_2 = 10 \text{ N})$$

∴ $$\mu = \frac{1.5}{10} = 0.15$$

where μ is the co-efficient of friction

When an additional weight 0.5 N is added to the hanging weight 1.5 N, the system stars moving with an acceleration '$a$'

(i) Acceleration of the two weights

Using equation (6.11), we have

$$a = \frac{g\ (W_1 - \mu W_2)}{(W_1 + W_2)} = \frac{9.81(2.0 - 0.15 \times 20)}{(20+10)} = \frac{9.81 \times 0.5}{12} = \mathbf{0.408\ m/s^2.\ Ans.}$$

(ii) Tension in the sting

Using equation (6.12) we have

$$T = \frac{W_1 W_2\ (1+\mu)}{(W_1 + W_2)} = \frac{2 \times 10 \times (1+0.15)}{(2+10)} \qquad (\because \mu = 0.15)$$

$$= \frac{20 \times 1.15}{12} = \mathbf{1.916\ N.\ Ans.}$$

**6.7.2. Motion of Two Bodies Connected by a String when One Body is Lying on Inclined Plane and the other is Hanging Free in Air**

**1. First case when the inclined surface is smooth**

Fig. 6.17 shows two bodies of weights $W_1$ and $W_2$ connected by a light inextensible string, which passes over a smooth and weightless pulley. The weights $W_2$ is placed on the horizontal, whereas the weight $W_1$ is hanging free in air.

As the inclined plane is smooth and hence the friction between the weight $W_2$ and the inclined plane will be neglected. When the weight $W_1$ is moving downwards, the weight $W_2$ will be moving upwards along the inclined plane. The velocity and acceleration of the weight $W_1$ will be same as that of weight $W_2$. Since the pulley is smooth and string is light and inextensible, the tension* in the string on both sides of the pulley will be same.

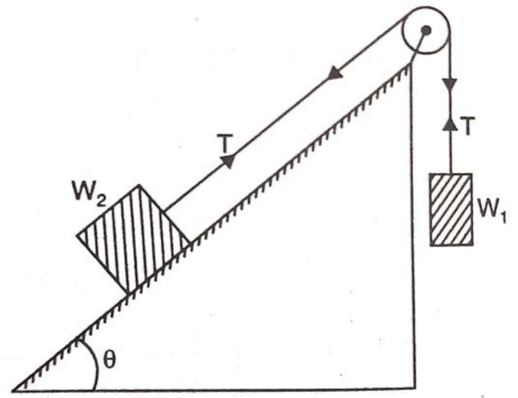


**Fig. 6.17**

Let $a$ = Acceleration of the system i.e., acceleration of weight $W_1$ as well as acceleration of weight $W_2$.

T = Tension in the string,

θ = Inclination of the inclined plane.

Consider the motion of weight $W_1$. The weight $W_1$ is moving downwards with an acceleration '$a$'. The forces acting on $W_1$ are: (i) its weight $W_1$ acting downwards, and (ii) tension T acting upwards.

∴ Net downwards force = $W_1 - T$

But net downwards force = mass × acceleration.

$$\therefore \qquad (W_1 - T) = \frac{W_1}{g} \times a \qquad \left(\because \text{mass} = \frac{W_1}{g}\right) \qquad \ldots(i)$$

**Now consider the motion of weight $W_2$.** The weight $W_2$ is moving upwards along the inclined plane with an acceleration $a$. The forces acting on $W_2$ along the plane are shown in Fig. 6.18. They are:

(i) $W_2 \sin\theta$ downwards and
(ii) Tension T upwards

∴ Net force acing on $W_2$ along the plane in the upwards direction

$$= T - W_2 \sin \theta$$

But net force = mass × acceleration.

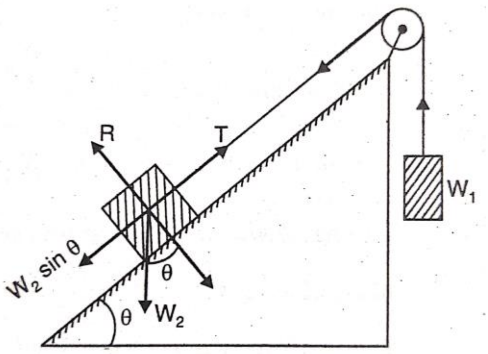


**Fig. 6.18**

$\therefore \qquad T - W_2 \sin \theta = \frac{W_1}{g} \times a$

$$(\because \text{ mass} = \frac{\text{weight}}{g}) \quad \ldots(i)$$

Adding equations (i) and (ii)

$$W_1 - W_2 \sin \theta = \frac{W_1}{g} \times a + \frac{W_2}{g} \times a$$

$$= \frac{a}{g} (W_1 - W_2) \text{ m/s}^2$$

Hence equation (6.13) is used for finding the acceleration of the system.

To find the value of tension T', in the string, the value of a from equation (6.13) is substituted in equation (i),

$$\therefore \qquad W_1 - T = \frac{W_1}{g} \times g \ \frac{(W_1 - W_2 \sin \theta)}{(W_1 + W_2)}$$

$$= \frac{W_1(W_1 - W_2 \sin \theta)}{(W_1 + W_2)} \qquad \text{(cancelling 'g')}$$

or

$$T - W_1 - \frac{W_1(W_1 - W_2 \sin \theta)}{(W_1 + W_2)} = W_1 \left[1 - \frac{(W_1 - W_2 \sin \theta)}{W_1 + W_2}\right]$$

$$= W_1 \left[\frac{W_1 + W_2 - W_1 + W_2 \sin \theta)}{W_1 + W_2}\right] = \frac{W_1[W_2 - W_2 \sin \theta]}{(W_1 + W_2)}$$

$$= \frac{W_1 W_2 (1 + \sin \theta)}{W_1 + W_2} \qquad \ldots(6.14)$$

Equation (6.14) is used for finding tension T in the string.

**2. Second case when the inclined surface is rough**

As the surface of the inclined plane is not smooth, hence a force of friction equal to μR will be acting on the weight $W_2$ in the opposite direction of motion of weight $W_2$ as shown in Fig. 6,19.

Let $\quad \mu$ = Co-efficient of friction between the weight $W_2$ and inclined surface,

θ = Angle of the inclination of the plane

$a$ = Acceleration of the system,

T = Tension in the string,

R - Normal reaction acting on $W_2$

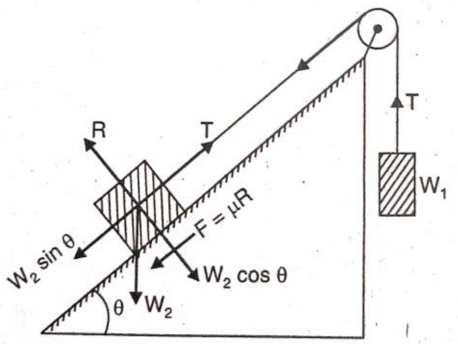


**Fig. 6.19**

The forces acting on the weight $W_2$ are shown in Fig. 6.19. Equating the forces normal to the plane, we get

$$R = W_2 \cos\theta$$

∴ Friction force, $F = \mu R$

$$= \mu W_2 \cos\theta \qquad \text{...(i)}$$

Consider the motion of weight $W_1$ is moving downwards with an acceleration $a$.

The net downward force acting on $W_1 = (W_1 - T)$.

Mass of weight $W_1 = \dfrac{W_1}{g}$

But net force = mass × acceleration.

$$\therefore \quad (W_1 - T) = \frac{W_1}{g} \times a \qquad \text{...(ii)}$$

Now consider the motion of weight $W_2$. The weight $W_2$ is moving upwards along the inclined plane with an acceleration a. The net upward force along the inclined plane acting on weight $W_2$

$$= T - W_2 \sin\theta - \mu R$$

$$= T - W_2 \sin\theta - \mu W_2 \cos\theta \qquad [\therefore \text{ From (i) } \mu R = \mu W_2 \cos\theta]$$

Mass of weight $W_2 = \dfrac{W_1}{g}$

Using, Net force = mass × acceleration.

$$\therefore \quad T - W_2 \sin\theta - \mu W_2 \cos\theta = \frac{W_1}{g} \times a \qquad \text{...(iii)}$$

Adding equation (ii) and (iii) we get

$$W_1 - W_2 \sin\theta - \mu W_2 \cos\theta = \frac{W_1}{g} \times a + \frac{W_2}{g} \times a = \frac{a}{g}[W_1 + W_2]$$

$$\therefore \quad a = \frac{g(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{(W_1 + W_2)} \qquad \text{...(6.15)}$$

Equation (6.15) is used for finding acceleration of the system. To find tension T in the string, the value of '$a$' from equation (6.15) is substituted in equation (ii).

$$\therefore \quad (W_1 - T) = \frac{W_1}{g} \times \frac{g(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{(W_1 + W_2)}$$

$$= \frac{W_1(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{(W_1 + W_2)}$$

$$\therefore \quad T - W_1 - \frac{W_1(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{(W_1 + W_2)}$$

$$= W_1 \left[1 - \frac{(W_1 - W_2 \sin\theta - \mu W_2 \cos\theta)}{(W_1 + W_2)}\right]$$

$$= W_1 \left[\frac{W_1 + W_2 - W_1 + W_2 \sin\theta + \mu W_2 \cos\theta)}{(W_1 + W_2)}\right]$$

$$= \frac{W_1}{(W_1 + W_2)} (W_1 + W_2 \sin\theta + \mu W_2 \cos\theta)$$

$$= \frac{W_1 W_2}{(W_1 + W_2)} (1 + \sin\theta + \mu \sin\theta) \qquad ...(6.16)$$

Equation (6.16)is used for finding tension in the sting.

**Problem 6.23.** Two bodies of weights 40 N and 15 N are connected to the two ends of a light inextensible string, which passes over a smooth pulley. The weight 40 N is placed on a smooth inclined plane, while the weight 15 N is hanging free in air. If the angle of the plane is 15°, determine:

(i) acceleration of the system, and
(ii) tension in the string. Take g = 9.80 $m/s^{-2}$

**Sol.** Given :

| | |
|---|---|
| Weight placed on inclined plane, | $W_2 = 40$ N |
| Weight hanging free in air, | $W_1 = 15$ N |
| Angle of inclination, | $\theta = 15°$ |
| Acceleration due to gravity, | $g = 9.80\ m/s^{-2}$ |

Let $a$ = Acceleration of the system
T = Tension in the string.

The inclined surface is smooth. Hence the acceleration and tension are obtained by using equations (6.13) and (6.14).

(i) Using equation (6.13) for acceleration,

$$a = \frac{g(W_1 - W_2 \sin\theta)}{(W_1 + W_2)}$$

$$= \frac{9.80(15 - 40 \sin 15°)}{(15 + 40)} = \frac{9.80(15 - 40 \times 0.2588)}{55}$$

$$= \frac{9.80 \times 4.684}{55} = \mathbf{0.828\ m/s^{-2}.\ Ans.}$$

(ii) Using equation (6.14) for tension,

$$T = \frac{W_1 - W_2\,(1 + \sin\theta)}{(W_1 + W_2)}$$

$$= \frac{15 \times 40(1 + .2588)}{(15 + 40)}$$

$$= \frac{15 \times 40 \times 12588}{55} = \mathbf{13.732\ N.\ Ans.}$$

## 6.8 LAW OF CONSERVATION OF MOMENTUM

It states that if the resultant of the external forces acting on a system is zero, the momentum of the system remains constant. This means that the total momentum of the system before collision is equal to the total momentum of the system after collision. The system may consist of one body or two bodies or more.

Impact means the collision of two bodies which occurs in a very small interval of time and during which the two bodies exert very large force on each other.

This large force is known as impulsive force. And the product of this impulsive force and small interval of time is known as impulse.

The two bodies A and B are moving in a horizontal line before collision with velocities $u_1$ and $u_2$ in the same direction i.e., along $x$-axis as shown in Fig. 6.20 (a). if $u_1 > u_2$ the body A will strike the body B and collision will take place. Let C is then point of collision of the two bodies as shown in Fig. 6.20 (b). The point C is also known as the point of contact. The line joining the centres of these two bodies and passing through the point of contact is known as **line of impact.** Hence here the line $O_1$-C-$O_2$ is called line of impact.

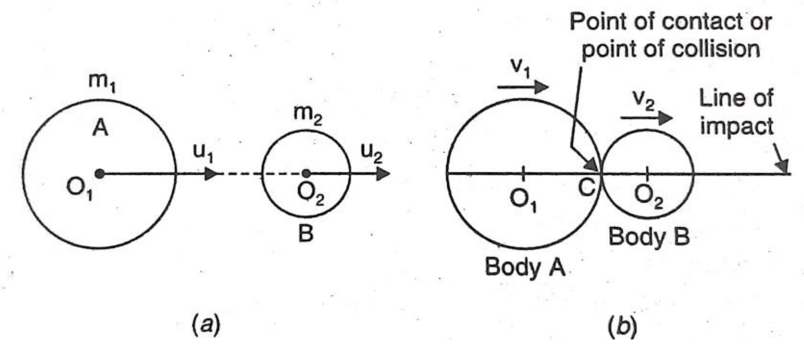


**Fig. 6.20**

The collision between two bodies is known as direct impact if the two bodies before impact, are moving along the line of impact.

The two bodies shown in Fig. 6.20 is having a direct impact.

Let $m_1$ = Mass of the body A

$u_1$ = Initial velocity of body A, i.e., the velocity of body A before collision along x-axis.

$v_1$= Final velocity of body A (after collision) along x-axis.

$m_2$, $u_2$ and $v_2$ are the mass of body B, velocity of body B before collision and velocity of the body B after collision along x-axis respectively.

The momentum of the body A before collision

$= \text{Mass} \times \text{Velocity} = m_1, u_1$, kg m/s.

The momentum of the body B before collision

$= m_2 \times u_2 = m_2u_2$ kg m/s.

∴ Total initial momentum (i.e., momentum before collision)

$= m_1u_1 + m_2u_2$, kg m/s ...(i)

Similarly, total final momentum (i.e., momentum after collision)

= Mass of body A × Final velocity of A + Mass of body B × Final velocity of body B

$= m_1v_1 + m_2v_2$ ...(ii)

But according to the law of conservation of momentum,

Total initial momentum = Total final momentum

or $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

**6.8.1. Problems Based on Conservation of Momentum**

Problem 6.24. Ball A of mass 1 kg moving with a velocity of 2 m/s, strikes directly on a ball B of mass 2 kg at rest. The ball A, after striking, comes to rest. Find the velocity of ball B after striking.

**Sol.** Given:

Mass of ball A, $m_1 = 1$ kg

Initial velocity of ball A, $u_1 = 2$ m/s

Mass of ball B $m_2 = 2$ kg

Initial velocity of ball B, $u_1 = 0$

Final velocity of ball A, $v_1 = 0$

This is a case of direct impact.

Let $v_2$ = Velocity of ball B after impact.

Total initial momentum $= m_1u_1 + m_2u_2 = 1 \times 2 + 2 \times 0 = 2$ kg m/s.

Total final momentum $= m_1v_1 + m_2v_2 = 1 \times 0 + 2 \times v_2 = 2v_2$ kg m/s.

According to the law of conservation of momentum,

Total initial momentum = Total final momentum

∴ $2 = 2 \times v_2$

∴ $v_2 = \frac{2}{2}\, 1$ **m/s. Ans.**

**Problem 6.25.** A body of mass 50 kg, moving with a velocity of 6 m/s, collides directly with a stationary body of mass 30 kg. If the two bodies become coupled so that they move on together after the impact, what is their common velocity.

**Sol.** Given :

Mass of first body, $m_1$ = 50 kg

Initial velocity of first body, $u_1$ = 6 m/s

Mass of second body, $m_2$ = 30 kg

Initial velocity of second body, $u_2$ = 0.

Total mass of two bodies = $(m_1 + m_2)$ = (50 + 30) = 80 kg.

Let V = Common velocity of the two bodies after impact.

Total momentum before impact = $m_1 u_1 + m_2 u_2$ = 50 × 6 + 30 × 0 = 300 kg m/s

Total momentum after impact = $m_1 + m_2$ × V = (50 + 30) × V = 80 V kg m/s

But total momentum before impact = Total momentum after impact

or 300 = 80 × V

∴ $V = \frac{300}{80}$ **3.75 m/s. Ans.**

**Problem 6.26**. A bullet of mass 50 gm is fired into a freely suspended target to mass 5 kg. On impact, the target moves with a velocity of 7 m/s along with the bullet in the direction of firing. Find the velocity of bullet.

**Sol.** Given :

Mass of bullet, $m_1$ = 50 gm = $\frac{50}{1000}$ V = $\frac{300}{80}$ = 0.05 kg

Mass of target, $m_2$ = 5 kg

Initial velocity of bullet = $u_1$

Initial velocity of target, $u_2$ = 0

Total mass of bullet and target = 5 + 0.05 = 5.05 kg

Final velocity of bullet and target = 7 m/s

Total initial momentum (i.e., momentum before impact)

$= m_1 \times u_1 + m_2 \times u_2 = 0.05 \times u_1 + 5 \times 0$

$= 0.05\, u_1$ kg m/s

Total final momentum (i.e., momentum after impact)

= Total mass × Common velocity = (5.05) × 7kg m/s

Equating the initial momentum to final momentum, we get

$$0.05\ u_1 = 5.05 \times 7 \therefore u_1 = \frac{5.05\ \times 7}{0.05} = \mathbf{707\ m/s.\ Ans.}$$

**Problem 6.27.** A ball of mass 20 kg moving with a velocity of 5 m/s strikes directly another ball of mass 10 kg moving in the opposite direction with a velocity of 10 m/s. Determine the velocity of first ball in terms of velocity of second ball after impact.

**Sol.** Giyen :

Mass of first ball, $m_1$ = 20 kg

Initial velocity of first ball, $u_2$ = 5 m/s

Mass of second ball, $m_2$ = 10 kg

Initial velocity of second ball, $u_2$ = - 10 m/s

(Negative sign is due to opposite direction)

Let $v_1$ = Velocity of first ball after impact

$v_2$ = Velocity of second ball after impact

Total momentum before impact

$= m_1u_1 + m_2u_2 = 20 \times 5 + 10 \times (-10)$

$= 100 - 100 = 0$

Total momentum after impact

$= m_1v_1 + m_2v_2 = 20v_1 + 10v_2$

Equating the total momentum after impact and before impact, we get

$$20v_2 + 10v_2 = 0 \text{ or } v_1 = \frac{-10v_2}{20} = -\frac{1}{2}v_2. \textbf{ Ans.}$$

## STUDENT ACTIVITY

**3.** Explain the terms : Momentum of a body.

**4.** Define and explain the Newton Laws of motion for linear motion.

## SUMMARY

1. The quantity of matter contained in a body known as mass of the body.
2. The weight of a body is defined as the force by which the body is attracted to wards the centre of earth. Weight of the body is given by

   Weight = Mass × g.

3. Momentum of a body is the product of the mass and its velocity.
4. The external force acting on a body is directly proportional to the rate of change of momentum in the same direction.

   $\therefore$ F = $m \times a$

   where $m$ = Mass of the body, and
   $a$ = Acceleration of the body.

5. A unit force is one, which produces unit acceleration on unit mass.
6. Newton is that force which acts on a body of mass one kilogram and produces an acceleration of 1 m/s$^2$ in the direction of force.
7. Dyne is that force which acts on a body or mass one gram and produces on acceleration of 1 cm/s$^2$ in the direction of force.
8. The relation between newton and dyne is given by
   1 N = $10^5$ dyne.
9. If the external force acts on the body in the direction of motion of the body, the body will have acceleration. But if the external force acts opposite to the direction of motion of the body, the body will have retardation.
10. A weight W is attached to one end of a string, which passes over a pulley of weight $W_0$, the other end of the string is attached to the periphery of the pulley. The acceleration with which the weight W moves downwards and tension (P) in the string are given by :

$$a = \frac{gW}{(W+\frac{W_0}{2})}$$

and $$P = \frac{WW}{(2W+W_0)}$$

11. Two weights $W_1$ and $W_2$ are connected to the two ends of a string, which passes over a rough pulley of weight $W_0$. The acceleration of the system and the tensions in the two parts of the strings are given by (If $W_1 > W_2$) :

$$a = \frac{G(W_1 - W_2)}{(W_1 + W_2 + \frac{W_0}{2})}, \quad T_1 = \frac{W_1[2W_2 + \frac{W_0}{2}]}{(W_1 + W_2 + \frac{W_0}{2})}$$

and $$T_2 = \frac{W_2[2W_1 + \frac{W_0}{2}]}{(W_1 + W_2 + \frac{W_0}{2})}$$

# TEST YOURSELF

## (A) Theoretical problem

1. Define the terms: Mass of a body and weight of a body. What is the relationship between the two ?
2. Explain the terms : Momentum of a body.
3. Define and explain the Newton Laws of motion for linear motion.
4. Derive the relation, F = $ma$
   where $m$ = Mass, $a$ = Acceleration, and F = Force acting on a body.
5. State Newton's law of motion and explain them by giving an example in each case.
6. Define a unit force, Newton and dyne. What is the relationship between a newton and a dyne ?
7. Two weights $W_1$ and $W_2$ are connected by a light and inextensible string, passing over a smooth pulley. If $W_1 > W_2$, prove that the acceleration (a) of the system and tension in the string are given by
$$a = \frac{g(W_1 - W_2)}{(W_1 - W_2)} \text{ and } T = \frac{2W_1W_2}{(W_1 - W_2)}$$
8. Two weights $W_1$ and $W_2$ are connected by a light and inextensible string, which passes over a smooth pulley. The weight $W_2$ is placed on a smooth horizontal surface and weight $W_1$ is hanging free. Prove that the acceleration ($a$) of the system and tension (T) in the string are given by,
$$a = \frac{gW_1}{(W_1 - W_2)} \text{ and } T = \frac{W_1\, W_2}{(W_1 - W_2)}$$

## (B) Numerical Problem

1. Find the force acting on a body of mass 100 kg and producing an acceleration of 2 m/s$^2$ in its direction.
   [**Ans.** 200 N]
2. A force of 450 N acts on a body having a mass of 150 kg for 5 seconds. If the initial velocity of the body is 10 m/s, determine :
   (i) acceleration produced in the direction of force, and
   (ii) distance moved by the body in 4 seconds.
   [**Ans.** (i) 3 m/s$^2$, (ii) 64 m]
3. The weight of a body on earth is 490 N. If the acceleration due to gravity on earth = 9.8 m/s$^2$, what will be the weight of the body on :
   (i) the moon where gravitational acceleration is 1.5 m/s$^2$, and
   (ii) the sun, where gravitational acceleration is 300 m/s$^2$.
   [**Ans.** (i) 75 N, (ii) 15000 NJ]
4. A force of 300 N acts on a body of mass 150 kg for 30 seconds, If the initial velocity of the body is 25 m/s, determine the final velocity of the body, when the force :
   (i) acts in the direction of motion, and
   (ii) acts in the opposite direction of motion.
   [**Ans.** (i) 85 m/s, (ii) - 35 m/s]

**5.** A body of mass 20 kg falls on the muddy ground from a height of 39.2 m. The body penetrates into the ground. Find the distance through which the body will penetrate into the ground, if the resistance by the ground to penetration is constant and equal to 980 N. Take g = 9.8 $m/s^2$.

[**Ans.** 9.8 m]

**6.** A man weighing 585 N dives into a swimming pool from a tower of height 9.8 m. He was found to go down in water by 1.5 m and then started rising. Find the average resistance of water.

[**Ans.** 452 kgf]

**7.** A bullet weighing 80 gmf and moving with a velocity of 350 m/sec is fired into a log of wood and it penetrates to a depth of 15 cm. If the bullet moving with the same velocity, were fired into a similar piece of wood 10 cm thick, with what velocity would it emerge? Find also the force of resistance assuming it to be uniform.

[**Ans.** 202 m/s, 32666 N]

**8.** A car, moving on a straight level road, skidded for a total distance of 40 metres after the brakes were applied. Determine the speed of the car, just before the brakes were applied, if the co-efficient of friction between the car tyres and the road is 0.4. Take g = 9.80 $m/s^2$.

[**Ans.** 63.75 km/hr]

**9.** A lift carries a weight of 110 N and is moving with a uniform acceleration of 3 $m/s^2$. Determine the tension in the cables supporting the lift, when

(i) lift is moving upwards, and

(ii) lift is moving downwards. Take g = 9.80 $m/s^2$.

[**Ans.** 143.67 N, 76.34 N]

**10.** A lift has an upward acceleration of 1.5 $m/s^2$. What pressure will a man weighing 500 N exert on the floor of the lift ? What pressure would he exert if the lift had an acceleration of 1.5 $m/s^2$ downward ? Take g= 9.8 $m/s^2$.

[**Ans.** 576.5 N, 423.5 N]

**11.** An elevator weighs 2000 N and is moving vertically downwards with a uniform acceleration. Write the equation for the elevator cable tension. Starting from rest it travels a distance of 30 m during an interval of 12 seconds. Find the cable tension during this time. Neglect all other resistances to motion. What are the limits of cable tension ?

[**Ans.** 1915.2 N, at $f$ = 0, T = 2000 and at $f$ = 9.81, T = 0]

**12.** An elevator weighing 6000 N is ascending with an acceleration of 2 $m/s^2$. During this ascent its operator whose weight is 600 N is standing on the scales placed on the floor. What is the scale reading ? What will be the total tension in the cables of the elevator during this motion ?

[**Ans.** 722.3 N, 7945.5 N]

**13.** Two bodies of weight 60 N and 40 N are connected to the two ends of a light in extensible string. The string is passing over a smooth pulley. Determine :

(i) the acceleration of the system, and

(ii) the tension in the string. Take g= 9.80 $m/s^2$

[**Ans.** 1.96 $m/s^2$, 48 N]

**14.** Two bodies of weight 40 N and 20 N are connected to the two ends of a light in extensible string, passing over a smooth pulley. They weight of 40 N is placed on a smooth horizontal surface while the weight of 20 N is hanging free in air. Find:

(i) the acceleration of the system, and

(ii) the tension in the string. Take g = 9.81 m/sec$^2$.

[**Ans.** 3.27 m/s$^2$, 13.33 N]

# 7 SIMPLE MACHINES

**LEARNING OBJECTIVES**

- Concept of Machine
- Mechanical Advantage, Velocity Ratio and Efficiency of a Machine
- Law of a Machine
- Lover
- Simple Wheel and Axle
- Single Purchase Crab Winch
- Pulleys
- Screw-Jack

## 7.1 CONCEPT OF MACHINE

A machine is a device which is capable of doing useful work. It receives the energy in some available form and uses that energy for doing a useful work.

**Lifting machines** are those machines, which are used for lifting loads. The force (or effort) is applied at one point of the machine and weight (or load) is lifted at the other point of the machine. The examples of lifting machine are lever, screw jack, inclined plane etc.

## 7.2 MECHANICAL ADVANTAGE, VELOCITY RATIO AND EFFICIENCY OF A MACHINE

To define the technical terms, used with a lifting machines, let us consider a machine which an effort $P$ is applied and a load $W$ is lifted.

Let $y$ = Distance moved by effort $P$

$x$ = Distance moved by the load $W$

Then the terms, and with the lifting machines are defined as:

### 7.2.1 Input of a Machine.

Input of a machine is defined as the work done on the machine. But the work is done on the machine by the effort. Hence the product of effort and distance moved by the effort gives the input of the machine.

Mathematically, input of a machine is given as,

Input = Effort × Distance moved by the effort = $P \times y$ ...(7.1)

### 7.2.2 Output of a Machine.

Output of a machine is defined as the actual work done by the machine. As machine is used for lifting load, hence work done by the machine is equal to the product of the load lifted and the distance through which load is lifted. Mathematically,

Output of the machine = Load × Distance through which load is lifted
=W × $x$ .... (7.2)

### 7.2.3. Efficiency of a Machine (η).

Efficiency of a machine is defined as the ratio of output of the machine to the input of the machine. It is denoted by the symbol η. Thus mathematically,

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{Wx}{Py} \qquad \text{.... (7.3)}$$

It is generally expressed as a percentage *i.e.,*

$$\eta = \frac{\text{Output}}{\text{Input}} = 100$$

### 7.2.4. Velocity Ratio (V.R.).

It is defined as the ratio of the distance moved by the effort to the distance moved by the load. It is denoted by the symbol V.R. Thus mathematically,

$$\text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{y}{x} \qquad \text{.... (7.4)}$$

### 7.2.5. Mechanical Advantage (M.A.).

It is defined as the ratio of the load or weight lifted to the effort applied. It is denoted by M.A. Thus mathematically,

$$\text{M.A.} = \frac{\text{Weight lifted}}{\text{Effort applied}} = \frac{W}{P} \qquad \text{.... (7.5)}$$

### 7.2.6. Ideal Machine.

If the friction in a machine is negligible, the machine is known as ideal machine. In ideal machine the efficiency is 100% and output is equal to input. Equating the input and output given by equations (7.1) and (7.2), we have for ideal machine.

$$P \times y = W \times x \text{ or } \frac{y}{x} = \frac{W}{P}$$

But from equation (7.4). $\frac{y}{x}$ = V.R. and from equation (7.5). $\frac{W}{P}$ = M.A. Hence for ideal machine.

$$\text{V.R.} = \text{M.A.} \qquad \text{.... (7.6)}$$

In actual practice, no machine is ideal. A part of the work done on the machine is always lost in overcoming friction and hence the work done on the machine is always greater than the work done by the machine.

### 7.2.7. Efficiency of a machine (η) in terms of mechanical advantage (M.A.) and velocity ratio (V.R.).

Efficiency of machine is given by equation (7.3) as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{W \times x}{P \times y} = \frac{W/x}{P/y}$$

$$= \frac{\text{M.A.}}{\text{V.R.}} \qquad (\because \frac{W}{P} = \text{M.A. and } \frac{x}{y} = \text{V.R.}) \qquad \text{... (7.7)}$$

**Problem 7.1.** An effort of 100 N is applied to a machine to lift a load of 900 N. The distance moved by the effort is 100 cm. The load is raised through a distance of 10 cm. Determine the mechanical advantage, velocity ratio and the efficiency of the machine.

**Sol.** Given :

| | |
|---|---|
| Effort applied | $P$ = 100 N |
| Load lifted | $W$ = 900 N |
| Distance moved by effort, | $y$ = 100 cm |
| Distance moved by load, | r = 10 cm |

(i) Mechanical advantage (M.A) is given by equation (7.5) as

$$\text{M. A.} = \frac{W}{P} = \frac{900}{100} = \textbf{9.0. Ans.}$$

(ii) Velocity ratio (V.R.) is given by equation (7.4) as

$$\text{V. R.} = \frac{x}{y} = \frac{100}{10} = \textbf{10.0. Ans.}$$

(iii) Efficiency (η) is given by equation (7.7) as

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{9.0}{10} = 0.9 \text{ or } 0.9 \times 100 = \textbf{90\% Ans.}$$

**Second Method**

The efficiency of the machine can also be calculated as:

Input to the machine = Effort applied × Distance moved by the effort

$$= P \times y = 100 \times 100 \text{ N cm.}$$

Output of the machine = Load × Distance moved by the load

$$= 900 \times 10 = 9000 \text{ N cm.}$$

∴ Efficiency, $$\eta = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{9000}{100 \times 100} \times 100 = \mathbf{90\%\ Ans.}$$

**Problem 7.2.** An effort of 500 N is applied through a distance of 5 m to a lifting machine to raise a load through a distance of 50 cm. If the efficiency of the lifting machine is 80%, determine.

(i) Load lifted by the machine,
(ii) Mechanical advantage.
(iii) Velocity ratio

**Sol.** Given:

Effort applied, $P = 500$ N
Distance moved by effort, $y = 5.0$ m
Distance moved by load, $x = 50$ cm $= 0.50$ m.
Efficiency, $\eta = 80\%$ or $\frac{80}{100} \times 0.8$.

(i) Let $W$ - Load lifted by the machine in Newton

Now $$\eta = \frac{\text{Output}}{\text{Input}}$$

But Output = Load × Distance moved by load

$$= W \times 0.50 \text{ Nm}$$

Input = Effort × Distance moved by effort

$$= 500 \times 5.0 = 2500 \text{ Nm}$$

∴ $$\eta = \frac{W \times 0.50}{2500} \text{ or } 0.8 = \frac{W \times 0.50}{2500}$$

∴ $$W = \frac{2500 \times 0.8}{0.5} = \mathbf{400\ N.\ Ans.}$$

(ii) Mechanical advantage (M.A.) is given by equation (7.5) as

$$\text{M.A.} = \frac{W}{P} = \frac{4000}{500} = \mathbf{8.0.\ Ans.}$$

(iii) Velocity ratio (V.R.) is given by equation (9.4.) as

$$\text{V. R.} = \frac{y}{x} = \frac{5.0}{0.50} = \mathbf{10.0.\ Ans.}$$

**Problem 7.3.** An effort of 20 N is applied to a machine to lift a load of 900 N The distance moved by the effort is 2.40 m und by the load the distance moved is 4 cm.

Determine:
(i) Mechanical advantage of the machine,
(ii) Velocity ratio of the machine,
(iii) Efficiency of the machine.

**Sol.** Given:

Actual effort, P = 20 N
Load lifted W = 900 N
Distance moved by effort, y = 2.40 m
Distance moved by load, $x$ = 4 cm = 0.04 m.

(i) Mechanical advantage (M.A.) is given by equation (7.5) as

$$\text{M. A.} = \frac{W}{P} = \frac{900}{20} = \mathbf{45.0.\ Ans.}$$

(ii) Velocity ratio (V.R.) is given by equation (7.4) as

$$\text{V. R.} = \frac{y}{x} = \frac{2.40}{0.04} = \mathbf{60.0.\ Ans.}$$

(iii) Using equation (9.7) for efficiency,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{45.0}{60.0} = \frac{3}{4} = 0.75 \text{ or } 0.75 \times 100 = \mathbf{75\%\ Ans.}$$

**Problem 7.4.** The efficiency of a lifting machine is 70% when an effort of 10 N is required to raise a load of 500 N. Determine the mechanical advantage and velocity ratio of the machine.

**Sol.** Given

Efficiency, $\eta = 70\% = \frac{70}{100} = 0.70$
Effort, P = 10 N
Load, W = 500 N

Mechanical advantage is given by equation (7.5) as

$$\text{M. A.} = \frac{W}{P} = \frac{500}{10} = \mathbf{50.\ Ans.}$$

Using equation (7.7), we get

$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

$\therefore \qquad \text{V. R.} = \frac{\text{M.A.}}{\eta} = \frac{50}{0.7} = \mathbf{71.43.\ Ans.}$

## 7.3 LAW OF A MACHINE

The law of a machine is defined by an equation which gives the relationship between the effort required to raise the corresponding load. This is obtained by drawing a graph between the efforts and the corresponding loads lifted by the efforts.

Fig. 7.1 shows the graph between effort and the load. The various values of the efforts required to raise the corresponding loads are plotted. Then a straight line is drawn as shown in Fig. 7.1. For an ideal machine the straight line will pass through the origin. But for an actual machine, the straight line will intercept the y-axis as shown in Fig. 7.1. The intercept OA = Effort applied to the machine to overcome friction. If the effort applied is less than OA, the load will not be lifted. The law of a machine is given mathematically as:

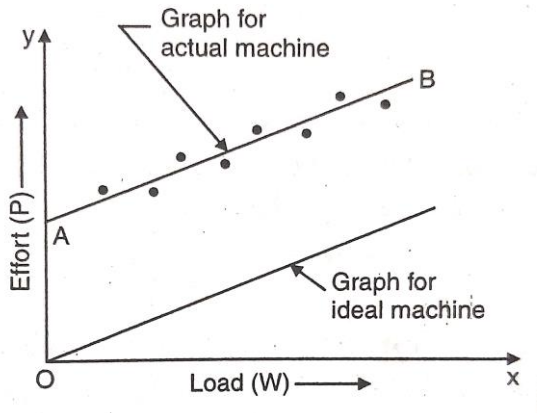


**Fig. 7.1** Law of machine

$$y = mx + c$$

where $y$ = Effort = $P$

$x$ = Load = $W$ and $c$ = constant

$\therefore$ The law of machine becomes $P = mW + c$ ...(7.8)

In equation (7.10)

$m$ = Slope of the line AB and is equal to a constant which is known co-efficient of friction.

c = Intercept of the line on $Y$-axis and is equal to the effort required to overcome friction.

Problem 7.5. In a lifting machine, an effort of 15 N raised a load of 770 N. What is the mechanical advantage ? Find the velocity ratio if the efficiency at this load is 60%.

If on the machine an effort of 25 N raised a load of 1320 N, what is the efficiency?

**Sol.** Given:

**First Case,**

Effort, $P_1$, = 15 N

Load, $W_1$ = 770 N

Efficiency, $\eta$ = 60% = 0.60.

Using equation (7.5),

$$\text{M. A.} = \frac{W_1}{P_1} = \frac{700}{15} = \mathbf{51.33.\ Ans.}$$

Using equation (7.7), $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

$\therefore$ $$\text{V. R.} = \frac{\text{M.A.}}{\eta} = \frac{51.33}{0.60} = \mathbf{85.55.\ Ans.}$$

**Second Case,**

Effort, $P_2$ = 25N

Load, $W_2$ = 1320 N

The velocity ratio will be same as in the first case.

$\therefore$ V.R. = 85.33

M.A. in second case $= \frac{W_2}{P_2} = \frac{1320}{25} =$ 52.80.

Efficiency is given by equation (7.7) as

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{52.80}{85.33} = 0.617 = \mathbf{61.7\%\ Ans.}$$

**Problem 7.6.** Find the law of a machine in which an effort of 15.6 N raised a load of 70 N and an effort of 19.5 N raised a load of 90 N. Find what effort is required to lift a load of 100 N?

**Sol.** Given

Effort $P$ = 15.5 N and load $W$ = 70 N

Effort $P$ = 19.5 N and load $W$ = 90 N

The law of the machine is given by equation (7.8), as

$$P = mW + C$$

Substituting the values of $P$ and Win the above equation,

$$15.5 = m \times 70 + C \quad \ldots\text{(i)}$$

And $$19.5 = m \times 90 + C \quad \ldots\text{(ii)}$$

Subtracting equation (i) from equation (ii). we get

$$4 = 20\ m$$

$\therefore$ $$m = \frac{4}{20} = \frac{1}{5} = 0.2$$

Substituting this value of min equation (i),

$$15.5 = 0.2 \times 70 + C = 14.0 + C$$

$\therefore$ $$C = 15.5 - 14.0 = 1.5$$

∴ The law of the machine becomes as

(i) $P$ = **0.2W + 1.5. Ans.**

(ii) Effort required to lift a load of 100 N

$$P = 0.2W + 1.5$$
$$= 0.2 \times 100 + 1.5 \qquad (\because W = 100 \text{ N})$$
$$= 20 + 1.5 = \mathbf{21.5.\ Ans.}$$

**Problem 7.7.** The efficiency of a machine is 80% when an effort of 15 N is required to lift a load of 130 N. Calculate the velocity ratio.

**Sol.** Given:

Efficiency, $\eta = 80\% = \frac{900}{20} = 0.8$

Effort, $P = 15$ N

Load, $W = 130$ N

Using equation (7.7), we get

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{W}{P}}{\text{V.R.}} \qquad (\because \text{M.A.} = \frac{W}{P})$$

or
$$0.80 = \frac{130}{\frac{15}{\text{V.R.}}} - \frac{130}{15 \times \text{V.R.}}$$

$$\text{V.R} = \frac{130}{15 \times 0.8} = \mathbf{10.833\ Ans.}$$

**Problem 7.8.** What load will be lifted by an effort of 12 N if the velocity ratio is 18 and efficiency of the machine at this load is 60%?

**Sol.** Given:

Effort, $P = 12$ N

Velocity ratio, V.R = 18

Efficiency, $\eta = 60\% = 0.60$

Frictional resistance of the machine = constant.

**First Part**

Let W = Load lifted by the effort $P = 12$ N

Then $\text{M.A.} = \frac{W}{P} = \frac{W}{12}$

Using equation (7.7), we have

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{W}{12}}{18} = \frac{W}{12 \times 18} \text{ or } 0.60 = \frac{W}{12 \times 18}$$

$$W = 0.60 \times 12 \times 18 = \mathbf{129.6\ N\ Ans.}$$

## 7.4 IMPORTANT LIFTING MACHINES

The following are the important lifting machines:

1. Lever
2. Simple wheel and axle.
3. Differential wheel and axle.
4. Single purchase crab winch.
5. Double purchase crab winch.
6. Pulleys:
   (a) First system of pulleys
   (b) Second system of pulleys
   (c) Third system of pulleys.
7. Screw jack:
   (a) Simple screw jack
   (b) Differential screw jack.

## 7.5 LEVER

Lover is a basic simple machine, which is used to lift heavy loads by applying a small force. It is a rigid straight bar which rests on a point, called fulcrum. The lever can also turnabout fulcrum.

Fig. 7.2 shows a lever which is resting on fulcrum point *O*. The effort is applied at point *B* and load (*W*) is lifted at point *A*.

Let $a$ = Distance of point A, at which load is applied, from fulcrum *O*.
$b$ = Distance of point B, at which effort is applied from fulcrum *O*.
$P$ = Effort applied at point *B*.
W = Weight lifted at point *A*.

When lever is in equilibrium, the resultant moment of all forces about point *O* should be zero i.e. $\sum M = 0$.

$$\therefore \quad W \times a = P \times b$$

or $$\frac{W}{b} = \frac{b}{a} \text{ or } P = \frac{a}{b} \times W$$

or $$W = \frac{b}{a} \times P$$

As the distance $b$ is more than distance $a$, hence load, $W$ will be more than effort $P$. Hence we can say that by applying a small force, a heavy weight can be lifted by lever.

## 7.6 SIMPLE WHEEL AND AXLE

Fig. 7.3 shows a simple wheel and axle which consists of two cylinders *A* and *B* of different diameters keyed to the same shaft. The bigger cylinder *A* is called the wheel, and the smaller cylinder *B* the axle.

A string is wound round the axle *B*, which carries the load *W* to be lifted. A second string is wound round the wheel *A*, to which effort is applied. The two strings are wound in the opposite directions. Hence a downward motion of *P* will raise the load *W*.

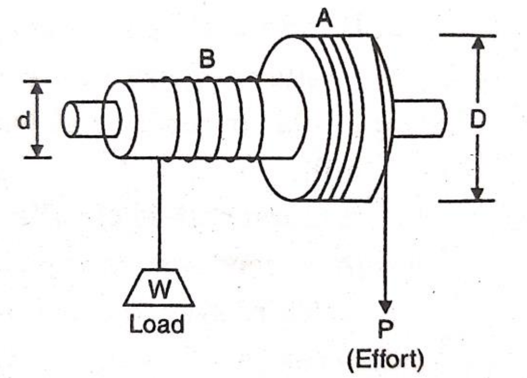


**Fig. 7.3**

Let $W$ = Load lifted,
$P$ = Effort applied,
$D$ = Diameter of wheel, and
$d$ = Diameter of axle.

As the wheel and axle are keyed to the same shaft, hence when wheel makes one revolution, the axle will also make one revolution.

Distance through which load moves in one revolution = $\pi d$
Distance through which effort moves in one revolution = $\pi D$

$$\text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{\pi D}{\pi d} = \frac{D}{d} \qquad \text{...(7.9)}$$

$$\text{M.A.} = \frac{W}{P} \text{ and } \eta = \frac{\text{M.A.}}{\text{V.R.}}$$

**Problem 7.9.** A weight of 48 N is to be raised by means of a wheel and axle. The axle is 100 mm diameter and wheel is 400 mm diameter. If a force of 16 N has to be applied to the wheel, find:
(i) Mechanical advantage,
(ii) Velocity ratio, and
(iii) Efficiency of the machine,

**Sol.** Given:
Weight $W$ = 48 N
Force $P$ = 16 N
Dia. of wheel $D$ = 400 mm
Dia. of axle, $d$ = 100 mm

(i) Mechanical advantage is given by,

$$\text{M.A.} = \frac{W}{P} = \frac{48}{16} = \mathbf{3.0.\ Ans.}$$

(ii) Velocity ratio is given by equation (7.9) as

$$\text{V.R.} = \frac{D}{d} = \frac{400}{100} = \mathbf{4.0.\ Ans.}$$

(iii) Efficiency of the machine is given by,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{3}{4} = 0.75 = \textbf{75\% Ans.}$$

### 7.6.1. Differential Wheel and Axle.

Fig. 7.4 shows a differential wheel and axle. The axle is made up of two cylinders *B* and *C* of different diameters. The wheel *A* and the axle *B* and *C* are keyed to the same shaft.

One string is wound round the wheel A to which effort *P* is applied. The Wheel second string is wound round the axle *B* and *C*. This string goes around a pulley to which the weight *W* is attached. This string is wound on the axle *B* and *C* in such a way that as the shaft rotate, the string unwinds on the axle *C* and winds at the same time on axle *B*, lifting the weight *W*. The two strings on the wheel *A* and on axle *C* must be wound in the same direction.

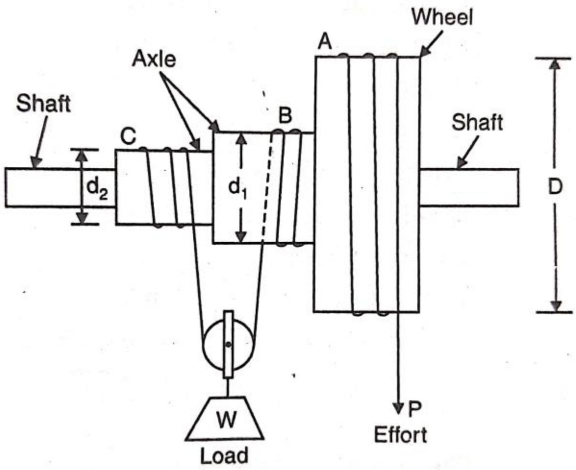


**Fig. 7.4**

When the string unwinds from the wheel *A*, the other string also unwinds from the axle *C*, but it winds on Fig. 7.4 the axle *B* as shown in Fig. 7.4.

Let $P$ = Effort applied,
$W$ = Weight lifted,
$D$ = Diameter of the wheel *A*,
$d_1$ = Diameter of the axle *B*, and
$d_2$ = Diameter of the axle *C*.

For one revolution of the wheel and axle:

Distance moved by effort = $\pi D$

Length of string that winds on the axle $B = \pi d_1$

Length of string that unwinds on the axle $C = \pi d_2$

As dia. of axle *B* is more than the dia, of axle *C*, hence $\pi d_1 > \pi d_2$

∴ Net length of string which will wound = $\pi d_1 - \pi d_2$

∴ Distance moved by weight $= \frac{1}{2}[\pi d_1 - \pi d_2]$

∴ $$\text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{\pi D}{\frac{1}{2}[\pi d_1 - \pi d_2]}$$

$$= \frac{2D}{d_1 - d_2} \qquad \text{...(7.10)}$$

and $\quad$ $\text{M.A.} = \frac{W}{P}$

$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

**Problem 7.10.** For a differential wheel and axle the diameter of wheel is 25 cm. The larger and smaller diameters of the differential axle are 10 cm and 9 cm respectively. An effort of 30 N is applied to lift a load of 900 N. Determine:

(i) Velocity ratio,
(ii) Mechanical advantage, and
(iii) Efficiency of the differential wheel and axle.

**Sol.** Given:

| | |
|---|---|
| Dia. of wheel, | $D$ = 25 cm |
| Large dia. of axle, | $d_1$ = 10 cm |
| Small dia. of axle, | $d_2$ = 9 cm |
| Effort applied | $P$ = 30 N |
| Lord lifted | $W$ = 900 N |

(i) Velocity ratio is given by equation (7.10) as

$$\text{V.R.} = \frac{2D}{d_1 - d_2} = \frac{2 \times 25}{10-9} = \mathbf{50.\ Ans.}$$

(ii) Mechanical advantage is given by,

$$\text{M.A.} = \frac{W}{P} = \frac{900}{30} = \mathbf{30.\ Ans.}$$

(iii) Efficiency is given by $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{3}{4} = 0.6 =$ **60% Ans.**

## 7.7 SINGLE PURCHASE CRAB WINCH

Fig. 7.5 shows a single purchase crab winch, which consists of an ‘effort axle' and a load axle. On effort axle, a small toothed wheel known as pinion is mounted. On load axle, a large toothed wheel known as spur wheel is mounted in such a way that the spur wheel meshes with the pinion. Both the axles are suitably mounted on a rigid frame as shown.

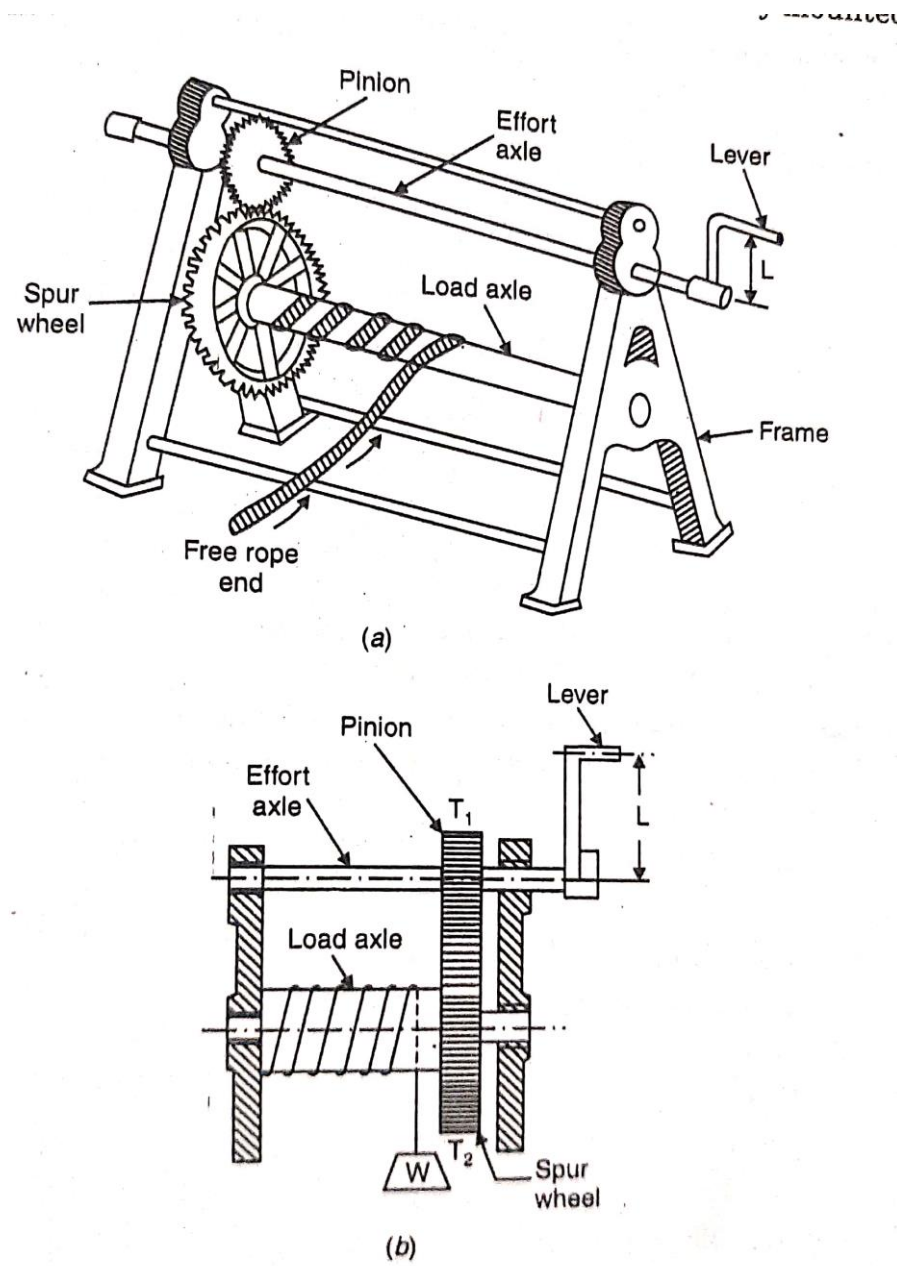


**Fig. 7.5**

A rope is fixed to the load axle and is wound a few turns round it. The free end of the rope carries the load *W*.

Effort is applied at the end of the lever which is fixed to the effort axle. By rotating the lever the pinion is rotated when in turn rotates the spur wheel and the rope is wound on the load axle, thus lifting the load attached to the free end of the rope.

Let $W$ = Load lifted,
$P$ = Effort applied,
$T_1$ = Number of teeth on the pinion,

$T_2$ = Number of teeth on the spur wheel,
L = Length of lever arm, and
D = Diameter of the load axle.

Consider one revolution of the lever arm.
Distance moved by effort in one revolution = $2\pi L$
When lever arm makes one revolution, the pinion also makes one revolution.

The spur wheel makes $\left(1 \times \frac{T_1}{T_2}\right)$ revolution. Also the load axle makes $\frac{T_1}{T_2}$ revolution.

∴ Distance moved by load in $\left(\frac{T_1}{T_2}\right)$ revolution

$$= \pi D \times \frac{T_1}{T_2}$$

$$\text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi L}{\pi D \times \frac{T_1}{T_2}} = \frac{2\pi L \times T_2}{D \times T_1} \qquad \text{...(7.11)}$$

$$\text{M.A.} = \frac{W}{P} \text{ and } \eta = \frac{\text{M.A.}}{\text{V.R.}}$$

**Problem 7.11.** The number of teeth on pinion and spur wheel of a single purchase crab winch are 10 and 100 respectively.

The diameter of load axle is 30 cm. The length of lever arm is also 30 cm. If an effort of 20 N is required to lift a load of 360 N on this machine, find:
(i) Velocity ratio, and
(ii) Efficiency of the machine.

**Sol.** Given:

| | |
|---|---|
| No. of teeth on pinion, | $T_1 = 10$ |
| No. of teeth on spur wheel, | $T_2 = 100$ |
| Dia. of load axle, | $D = 30$ cm |
| Length of lever, | $L = 30$ cm |
| Effort applied | $P = 20$ N |
| Load lifted, | $W = 360$ N |

(i) Velocity ratio is given by equation (7.11) as

$$\text{V.R.} = \frac{2\pi L \times T_2}{D \times T_1} = \frac{2 \times 30 \times 100}{30 \times 10} = \textbf{20. Ans.}$$

(ii) Efficiency is given by, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

W

where M.A. = $\frac{W}{P} = \frac{360}{20} = 18$

$\therefore \qquad \eta = \frac{18}{20} = 0.9 =$ **90% Ans.**

### 7.7.1. Double Purchase Crab Winch.

Fig. 7.6 shows a double purchase crab winch, in which velocity ratio is obtained in two stages by two pairs of gears. It consists of effort axle, load axle and intermediate axle.

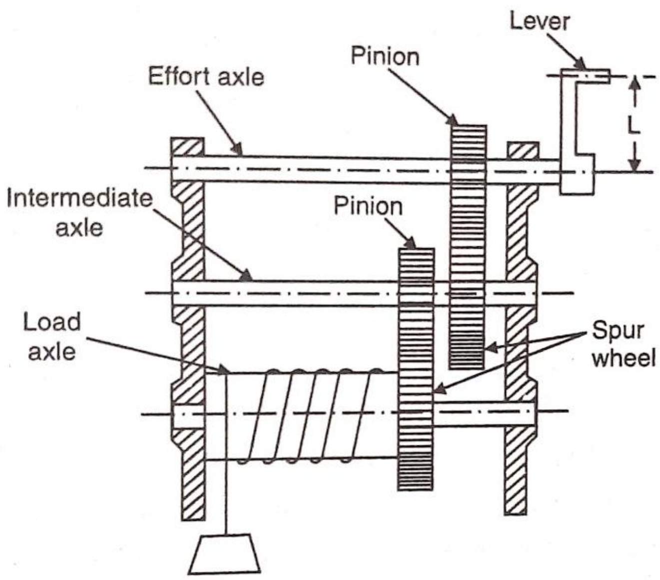


**Fig. 76**

On the effort axle, a pinion is mounted whereas on the load axle a spur wheel is mounted. On the intermediate axle a pinion and a spur wheel is mounted. The pinion of intermediate axle gears with the spur wheel of the load axle. And the spur wheel of the intermediate axle gears with the pinion of the effort axle.

The effort is applied at the end of the lever, which is fixed to the effort axle.

Let $W$ = Load lifted,
$P$ = Effort applied,
$T_1$ - Number of teeth on the pinion of effort axle,
$T_2$ = Number of teeth on the spur wheel of intermediate axle,
$T_3$ = Number of teeth on the pinion of intermediate axle,
$T_4$ = Number of teeth on the spur wheel of load axle,
$L$ = Length of lever, and
$D$ = Diameter of loud axle.

Consider one revolution of lever.
Distance moved by the effort = $2\pi L$
No. of revolution made by pinion of effort axle = 1
No. of revolution made by the spur wheel of intermediate axle

$$= \frac{T_1}{T_2}$$

No. of revolution made by the pinion of intermediate axle

$$= \frac{T_1}{T_2}$$

No. of revolution made by the spur wheel of load axle

$$= \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

∴ Distance moved by the load = $\pi D = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$

$$\text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi L}{\pi D \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}} = \frac{2\pi L}{D \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}}$$

$$= \frac{2L}{D} \times \frac{T_2}{T_1} \times \frac{T_4}{T_3} \qquad \ldots(7.12)$$

**Problem 7.12.** Find the V.R. and the load which can be lifted by an effort of 40 N if the efficiency of the double purchase crab is 50%. The specifications of this machine are:

Dia. of load axle (drum) = 20 cm
Length of lever = 80 cm
No. of teeth on the pinion of effort axle = 10
No. of teeth on the spur wheel of intermediate axle = 100
No. of teeth on the pinion of intermediate axle = 20
No. of teeth on the spur wheel of load axle = 200.

**Sol.** Given:
Effort = 40 N
Efficiency, $\eta$ = 50% = 0.5
Dia. of load axle, $D$ = 20 cm
Length of lever, $L$ = 80 cm

No. of teeth on pinion of effort axle, $T_1$ = 10
No. of teeth on spur wheel of intermediate axle, $T_2$ = 100
No. of teeth on pinion of intermediate axle, $T_3$ = 20
No. of teeth on spur wheel of load axle, $T_4$ = 200

(i) Velocity ratio is given by equation (7.12),

$$\text{V.R.} = \frac{2L}{D} \times \frac{T_2}{T_1} \times \frac{T_4}{T_3} = \frac{2 \times 80}{20} \times \frac{100}{10} \times \frac{200}{20} = \mathbf{800.\ Ans.}$$

(ii) Efficiency is given by, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

$\therefore$ $\quad$ M.A. = η × V.R = 0.5 × 800 = 400

But $\quad$ M.A. = $\frac{W}{P}$

$\therefore$ $\quad$ $\frac{W}{P}$ = 400 $\quad$ or $\quad$ $W$ = 400 × $P$

= 400 × 40 = 16,000 N = **16 kN. Ans.**

**Problem 7.13.** In a double purchase crab, the pinions have 15 and 20 teeth while the spur wheels have 45 and 40 teeth. The effort handle is 40 cm while the effective dis, of the drum is 15 cm. If the efficiency of the winch is 40% and load lifted is 250 N then what effort will be applied at the end of the handle.

**Sol.** Given:
No. of teeth on pinion of effort axle, $T_1$ = 15
No, of teeth on pinion of intermediate axle, $T_2$ = 20
No. of teeth on spur wheel of intermediate axle, $T_3$ = 45
No. of teeth on spur wheel of load axle, $T_4$ = 40
Length of effort handle, $L$ = 40 cm
Effective dia. of load axle, D = 15 cm
Efficiency, η = 40% 0.40
Load lifted = 250 N
Let $P$ = Effort applied.

The velocity ratio is given by equation (7.12) as

$$\text{V.R.} = \frac{2L}{D} \times \frac{T_2}{T_1} \times \frac{T_4}{T_3} = \frac{2 \times 40}{15} \times \frac{45}{15} \times \frac{40}{20} = 32$$

Now efficiency is given by, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

$$= \frac{\left(\frac{W}{P}\right)}{\text{V.R.}} \qquad \left(\because \quad \text{M.A.} = \frac{W}{P}\right)$$

$$= \frac{W}{P \times \text{V.R.}}$$

$\therefore$ $\quad$ $0.40 = \frac{250}{P \times 32}$

$\therefore$ $\quad$ $P = \frac{250}{0.40 \times 32}$ = **19.53 N. Ans.**

## 7.8 PULLEYS

A pulley is a wheel of metal or wood, with a groove around its circumference, to receive a rope or chain. While dealing with pulleys the following assumptions are made:

1. The weight of pulley is small as compared to the weight to be lifted and thus may be neglected.

2. The friction between the pulley and the rope in negligible and hence the tensions in the two sides of the rope, passing round the pulley, may be taken to be equal.

Fig. 7.7 shows a simple pulley with a block supporting the pulley. A rope is also passing round the circumference of the pulley. If the block is fixed then the pulley is known as fixed pulley. Fig. 7.7 shows a fixed pulley and if block is movable, then pulley is known as movable pulley, Fig. 7.8 shows a movable pulley.

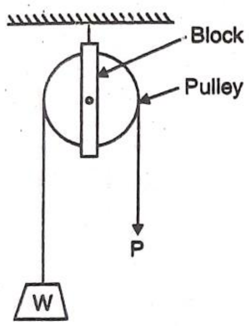


**Fig. 7.7**

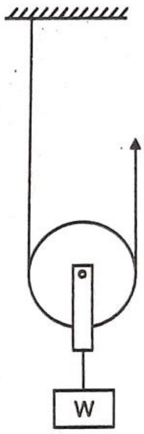


**Fig. 7.8**

**A single fixed pulley.** Fig. 7.9 shows a fixed pulley, in which effort is applied in any convenient direction. The weight $W$ is attached to one end of the string and effort is applied at the other end. In all these cases:

(i) P = W M.A. = $\left(\text{i. e., M. A.} = \text{W} = \frac{W}{P} = 1\right)$

(ii) Distance moved by effort = Distance moved by load,

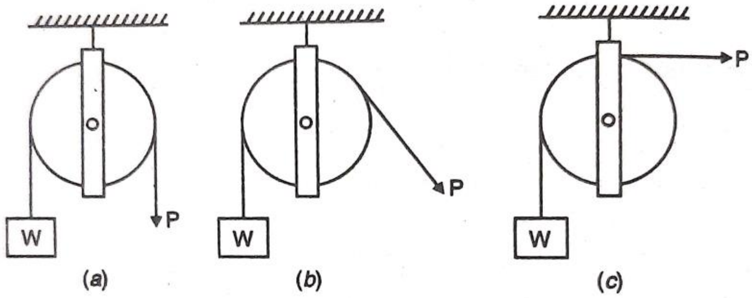


**Fig. 7.9**

**A movable pulley.** Fig. 7.10 shows a single movable pulley. One end of the string is attached to the fixed support and effort is applied at the other end. The weight $W$ will be equally shared by the two portions of the string.

$$\therefore \qquad P = \frac{W}{2} \text{ or } \frac{W}{P} = 2$$

Hence here the M.A. is more than one. But the force cannot be applied easily. So to make force of application to be applied easily, a fixed pulley is introduced which will not increase the M.A., but will help the operator to use the force conveniently as shown in Fig. 7.11.

From Fig. 7.11, it is clear that to raise the load $W$ through a certain distance, the effort $P$ will have to traverse double the distance. Hence velocity ratio in this case will be equal to 2.0.

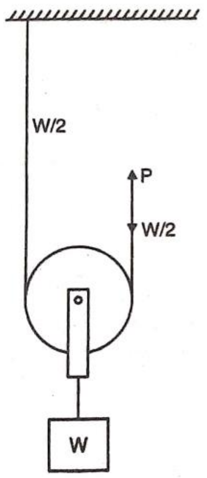


**Fig. 7.10**

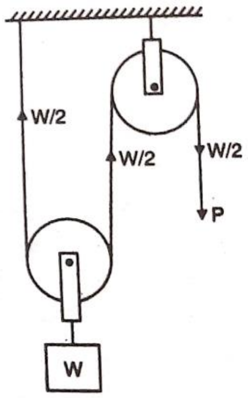


**Fig. 7.11**

**System of pulleys.** The pulleys are generally used in certain combinations to obtain a higher mechanical advantage and efficiency. The following three system of pulleys are commonly used:

(a) First system of pulleys,
(b) Second system of pulleys, and
(c) Third system of pulleys.

## 7.8.1. First System of Pulleys.

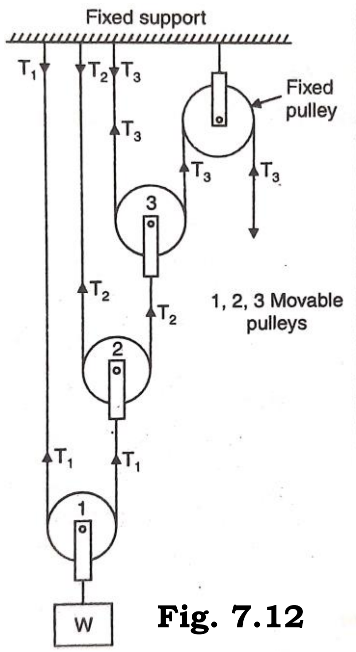


**Fig. 7.12**

Fig. 7.12 shows the first system of pulley. In this system a number of movable pulleys are used to give a greater mechanical advantage and convenience in application of effort is attained by using last of all a fixed pulley. In this system one end of each string is tied to the fixed support while the other end passing round the periphery of the bottom pulley is fastened to the block of the next higher pulley The load attached to the bottom-most pulley, whereas the effort is applied to the end of the string passing over a fixed pulley.

**Velocity ratio.** Let the string be pulled down by the effort through a distance $y$. The pulley 3, is supported by two segments of the string, hence each segment shortens by an equal amount $y/2$. The centre of pulley 3, therefore moves up by a vertical distance equal to $y/2$. The upward movement of pulley 3, moves the centre of pulley 2 by $\frac{1}{2}$ of $\frac{y}{2}$ or $\frac{y}{2^2}$ Similarly, the centre of pulley 1, moves up by a distance = $\frac{1}{2}$ of $\frac{y}{2^2} = \frac{y}{2^2}$

But to the pulley 1, weight is attached. Hence distance through which weight is lifted $\frac{y}{2^2}$

$$\text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{y}{\left(\frac{y}{2^3}\right)}\ 2^3$$

If there are '$n$' movable pulleys, then

$$\text{V.R.} = 2n$$

$$\text{M.A.} = \frac{W}{P}$$

For ideal machine, η = 1 or 100% and hence

$$\text{M.R.} = \text{V.R.}$$

**Problem 7.14.** There are four movable pulleys in a system of pulleys of the first type. If a load of 1440 N is lifted by an effort of 100 N. find:

(i) efficiency of machine,
(ii) effort wasted in friction, and
(iii) load wasted in friction.

**Sol.** Given:

No. of movable pulleya, $n = 4$

Load $W = 1440$ N

Effort, $P = 100$ N.

(i) The efficiency is given by, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

where M.A. $= \frac{W}{P} = \frac{1440}{100} = 14.4$

and V.R. is given by equation (7.13) as

V.R. $= 2^n = 2^4 = 16$

$\therefore$ $\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{14.4}{16} = 0.9$ or **90%. Ans.**

(ii) Effort wasted in friction

For an ideal machine (or frictionless machine)

$\eta = 1$ or 100%

or M.A. = V.R

But V.R. = 16

$\therefore$ M.A. = 16

But M.A. $= \frac{W}{P^*}$ where $P^*$ is ideal effort

$\therefore$ $\frac{W}{P^*} = 16$

or $P^* = \frac{W}{16} = \frac{1440}{16} = 90$ N

But actual effort, P = 100 N

$\therefore$ Effort wasted in friction = Actual effort - Ideal effort

= 100 - 90 = **10 N. Ans.**

(iii) Load wasted in friction

Let $W^*$ = Ideal load

For ideal machine M.A. = V.R. = 16

or $\frac{W^*}{P} = 16$

or $W^* = 16 \times P$, where $P$ = Actual effort = 100 N

$\therefore$ $= 16 \times 100 = 1600$ N

$\therefore$ Load wasted in friction = Ideal load – Actual load

= 1600 – 1440 = **160 N. Ans**

## 7.8.2. Second System of Pulleys.

Fig. 7.13 (b) and (c) shows the second system of pulleys which consists of two blocks each containing a number of pulleys. The upper block is fixed to a fixed support whereas the lower block is movable. Both the blocks carry either equal number of pulleys or the upper block may have one pulley more than the lower one.

The same string is passed round all the pulleys. One end of the string is fixed to the lower block if the upper block has one pulley more than the lower one as shown in Fig. 7.13 (b)], and the other end of the string is free and effort is applied to this free end.

If both the blocks have the same number of pulleys, then one end of the string is fixed to the upper block as shown in Fig. 7.13 (c) and effort is applied to the other free end of the string. In both cases, the weight is attached to the lower block.

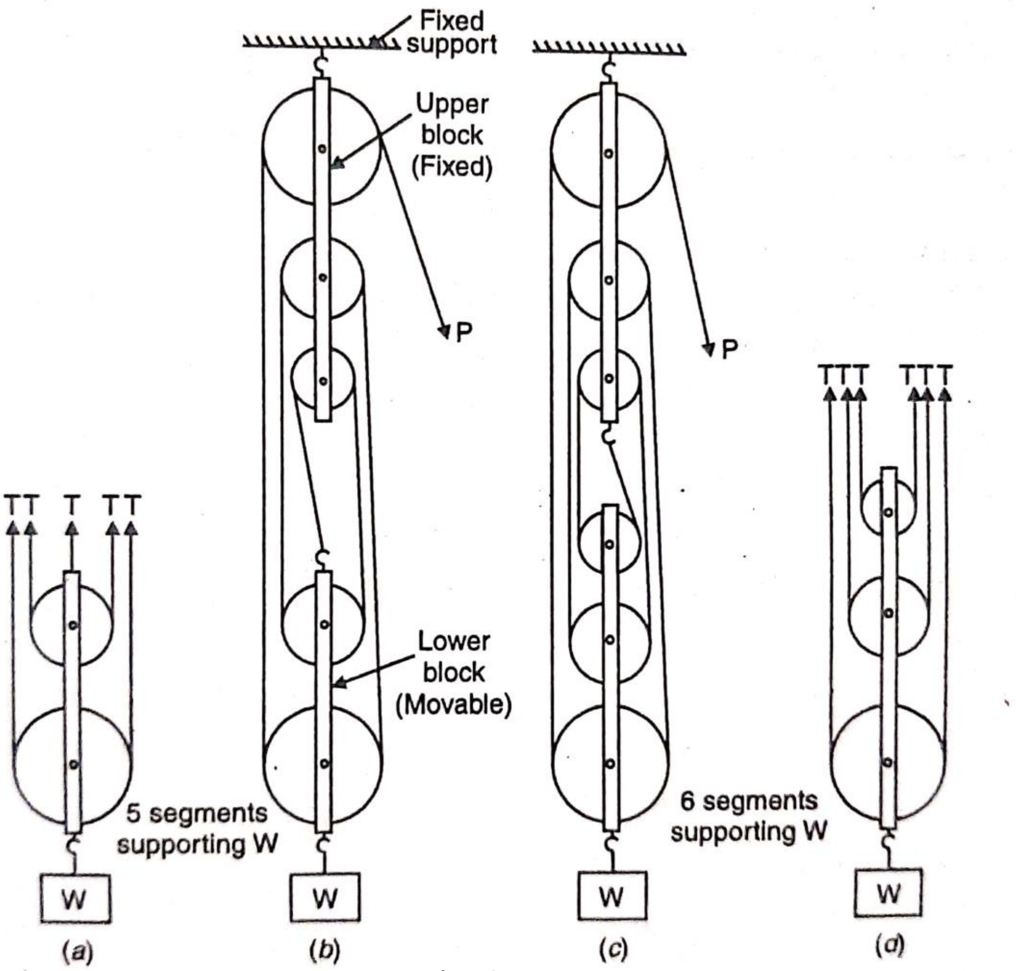


**Fig. 7.13**

The pulleys being smooth, the tension ($T$) in the string throughout will be equal to the applied effort $P$.

Let $n$ = number of segments supporting the lower block with weight $W$.

The force supporting the lower block = $n \times T$

$$= n \times P \qquad (\because T = P)$$

When lower block is in equilibrium, $W = n \times P$

or $$\frac{W}{P} = n$$

$\therefore$ $$\text{M.A.} = n$$

Since the system is ideal (i.e., without friction), hence

$$\text{M.A.} = \text{V.R.}$$

$$\text{V.R.} = n \qquad \ldots(7.14)$$

Where n = number of segments supporting the movable block or load

= always total number of pulleys in two blocks.

When weight of lower block is taken into consideration

Let $W$ = Weight to be lifted

$w$ = Weight of lower block

For the equilibrium of the lower block,

Forces supporting lower block = W + $w$

or $$n \times p = W \times w$$

or $$W = n \times P - w$$

or $$\frac{W}{P} = n - \frac{w}{P} \qquad \ldots(7.15)$$

The V R. of the system will be same and is equal to $n$.

$\therefore$ $$\text{V.R.} = n \qquad \ldots[7.15(A)]$$

where $n$ = number of segments supporting the movable block or load
= always total number of pulleys in two blocks. ...[7.15(B)]

**Problem 7.15.** A weight of 2000 N lifted by an effort of 600 N, by second system of pulleys having three pulleys in the upper block and two pulleys in the lower block. Find the efficiency of the system.

**Sol.** Given:

Weight, $W$ = 2000 N

Effort, P = 600 N

Total number of pulleys = 3 + 2 = 5

From equation (7.14), we know

$$\begin{aligned} \text{V.R.} &= n \\ &= \text{Number of segments supporting the movable block} \\ &= \text{Total number of pulleys in two blocks} \\ &= 5 \text{ (here)} \end{aligned}$$

Efficiency is given by, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

where $\text{M.A.} = \frac{W}{P}$

$$= \frac{2000}{600} = \frac{10}{3}$$

$\therefore$ Efficiency $= \frac{\left(\frac{10}{3}\right)}{5} = \frac{10}{3} \times \frac{1}{5} = 0.667 =$ **66.7%. Ans.**

### 7.8.3. Third System of Pulleys.

Fig. 7.14 shows the third system of pulley. In this system, several movable pulleys are arranged, keeping the top-most pulley as fixed. The number of strings are equal to the number of pulleys. One end of each string is attached to a common block, to which load is attached. The other end of each string, passing round the periphery of the pulley, is fastened to the block of the next lower pulley. The effort is applied to the free end of lower most pulley.

Neglecting friction and weight of pulleys, the equilibrium of the system gives

$$W = T_1 + T_2 + T_3 + T_4$$

But from equilibrium of pulleys 1, 2, 3 and 4

$$\begin{aligned} T^1 &= P \\ T^2 &= 2T^1 = 2P \\ T^3 &= 2T^2 = 2 \times 2P = 22P \\ T^4 &= 2T^3 = 2 \times 2^2T = 2^3P \\ \therefore \quad W &= P + 2P + 2^2P + 2^3P = P\,[1 + 2 + 2^2 + 2^3] \end{aligned}$$

$$\frac{W}{P} = 1 + 2 + 2^2 + 2^3$$

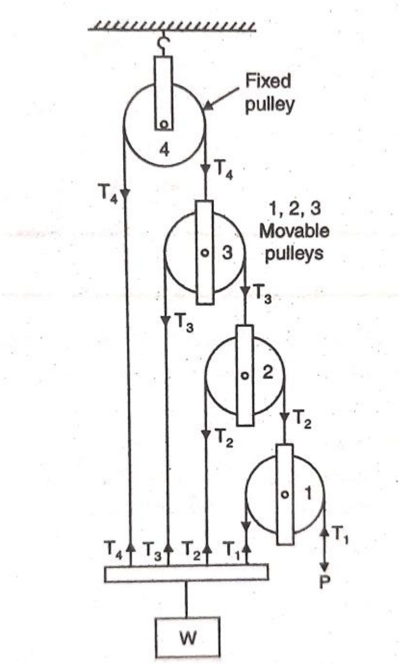


**Fig. 7.14**

If there are $n$ pulley = $\frac{W}{P}$ = 1 + 2 + $2^2$ + $2^3$ + .......... + $2^n - 1$

This is a geometrical progression,

∴ $$\frac{W}{P} = \frac{2^n - 1}{(2-1)} = 2^n - 1$$

For an ideal machine (i.e., frictionless machine)

$$\eta = 1 = \frac{\text{M.A.}}{\text{V.R.}}$$

∴ $$\text{V.R.} = \text{M.A.} = \frac{W}{P} = 2^n - 1$$

∴ $$\text{V.R.} = 2^n - 1 \qquad \text{...(7.16)}$$

**Problem 716.** There are four pulleys in a third system of pulleys. An effort of 160 N required to lift an unknown weight. If the efficiency of this machine is 75% find the weight lifted.

**Sol.** Given:
No. of pulleys, $n = 4$
Effort, $P = 160$ N
Efficiency $\eta = 75\% = 0.75$
Let $W$ = Weight lifted
Velocity ratio of the third system of pulley la given by equation (7.16)

$$\text{V.R.} = 2^n - 1 = 2^4 - 1 = 15$$

$$\text{M.A.} = \frac{W}{P} = \frac{W}{160}$$

Efficiency is given by, $\eta = \frac{\text{M.A.}}{\text{V.R.}}$

or $$0.75 = \frac{\left(\frac{W}{160}\right)}{15} = \frac{W}{160 \times 15}$$

$\therefore$ $W = 0.75 \times 160 \times 15 =$ **1800 N. Ans.**

# 7.9 SCREW-JACK

A screw-jack is a device used for lifting heavy weight or loads with the help of a small effort applied at its handle. The followings are two types of screw-jack:

(a) Simple screw-jack, and
(b) Differential screw-jack

## 7.9.1. Simple Screw-Jack.

Fig. 7.15 shows the simple screw-jack, which consists of a nut, a screw with square threads and n handle fitted to the head of the screw. The nut also forms the body of the jack.

The load to be lifted is placed on the head of the screw. At the end of the handle, fitted to the screw bead, an effort $P$ is applied in the horizontal direction to lift the load $W$.

Let $W$ = Weight placed on the screw head,
$P$ = Effort applied at the end of the handle,
$L$ = Length of handle,
$p$ = Pitch of the screw,

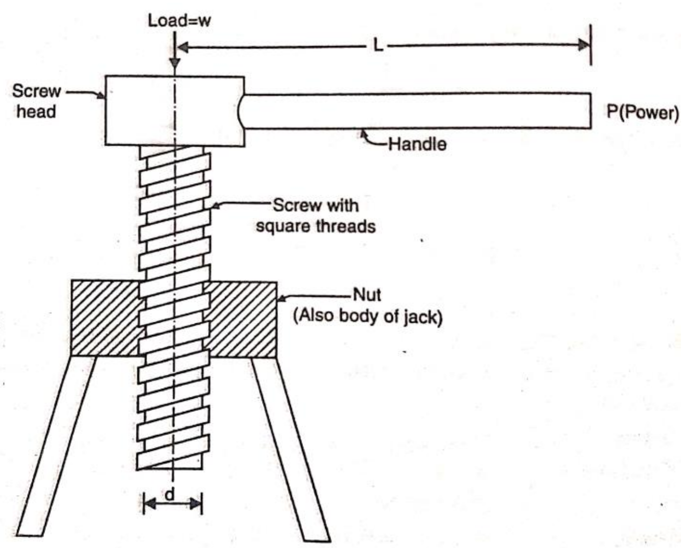


**Fig. 7.15** Simple screw-jack

When the handle is rotated through one complete turn, the screw is also rotated through one turn. Then the load is lifted by a height p (pitch of screw).

Distance moved by effort for one turn of the handle

$$= 2\pi L$$

$$\text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$= \frac{2\pi L}{P}$$

and mechanical advantage $= \dfrac{W}{P}$

$\therefore$ Efficiency, $\eta = \dfrac{\text{M.A.}}{\text{V.R.}} = \dfrac{\left(\frac{W}{P}\right)}{\left(\frac{2\pi L}{P}\right)}$

**Problem 7.17.** The efficiency of a screw-jack in 55%, when a load of 1500 N is lifted by an effort applied at the end of a handle of length 50 cm. determine the effort applied if the pitch of the screw thread is 1 cm.

**Sol.** Given:

Efficiency, $\eta = 55\% = \dfrac{55}{100} = 0.55$

Load lifted, $W = 1500$ N

Length of handle $L = 50$ cm $= 0.50$ m

Pitch of the screw, $p = 1$ cm $= 0.01$ m

Let $P$ = Effort applied

When the handle makes one complete turn, the load is lifted by a distance equal to pitch of the screw.

$\therefore$ Distance moved by load $= p = .01$ m

Distance moved by effort $= 2\pi L = 2\pi \times 0.50$ m

$$\therefore \quad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi L \times 0.50}{0.01} = 314.16$$

Mechanical advantage, M.A $= \dfrac{W}{P} = \dfrac{1500}{P}$

Using equation (7.7), we get

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{1500}{P \times 314.16} \qquad \text{or } 0.55 = \frac{1500}{P \times 314.16}$$

$$P = \frac{1500}{0.55 \times 314.16} = \mathbf{8.68\ N.\ Ans.}$$

## 7.9.2. Differential Screw-Jack.

Fig. 7.16 shows a differential screw-jack. The principle, on which this machine work is the same as that of any other differential machine i.e., action of one part of the machine is subtracted from the action of another part.

The differential screw is in two parts, *A* and *B*. Part *A* is threated both on inside and outside; whereas the part is threaded on the outside only. The external thread of a pear with the threads of the nut *C*, which form the body of the differential screw-Jack. The internal threads of A gear with the external threads of the screw *B*. Thus the part *A* behaves as a screw for the nut *C* and as a nut for the crew *B*.

The crew *B* does not rotate, but moves in vertical direction only, and carries the load. When the effort in applied at the lever, the crew. *A* rise up and simultaneously the screw *B* goes down. Thus the net life of the load in algebraic sum of the motion of the crew *A* and screw *B*.

Let $p_1$ = Pitch of the screw *A*
$p_2$ = Pitch of the screw *B*
$l$ = Length of the lever arm
$W$ = Load lined, and
$P$ = Effort applied to lift the lond, at the end of the lever.

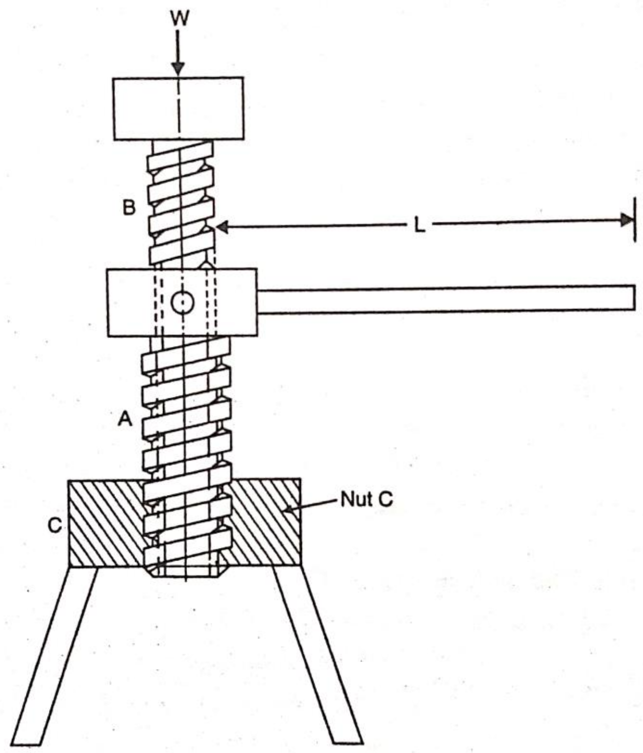


**Fig. 7.16**

Consider one revolution of the lever arm.
∴ Distance moved by the effort = $2\pi l$
Upward distance moved by $A$ = $p_1$
Downward distance moved by $B$ = $p_2$
Therefore the distance through which the load is lifted = $p_1 - p_2$

$$\therefore \qquad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi L}{p_1 - p_2}$$

$$\text{M.A} = \frac{W}{P}$$

$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

**Problem 7.18.** In a differential screw-jack, the screw threads have pitch of 10 mm and 7 mm. If the efficiency of the machine is 28%, find the effort required at the end of an arm 36 cm long to lift a load of 5 kN.

**Sol.** Pitch of larger screw. $p_1$ = 10 mm = 1 cm
Pitch of smaller screw $p_2$ = 7 mm 0.7 cm
Efficiency $\eta$ = 28% = 0.28
Length of handle $l$ = 36 cm
Weight, $W$ = 5 kN = 5000 N
Let $P$ = Effort required to lift the load.

Now, $$\text{V.R.} = \frac{2\pi L}{p_1 - p_2} = \frac{2\pi L \times 36}{1 - 0.7} = 754$$

and $$\text{M.A c} = \frac{5000}{P}$$

$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

or $$0.28 = \frac{\left(\frac{5000}{P}\right)}{754} = \frac{6.63}{P}$$

or **P = 23.7 N. Ans.**

# STUDENT ACTIVITY

**1.** What is the law of a machine?

**2.** What is a screw jack?

## SUMMARY

**1.** A machine is a device which is used for doing useful work. If the machine is used for lifting loads, the machine is known as lifting machine.

**2.** The work done on the machine is called input of the machine while the work done by the machine is known as output of the machine. The ratio of output to input is known as efficiency of the machine.

**3.** A machine is known as ideal machine if the efficiency of the machine is 100%. In that case input is equal to output of the machine.

**4.** Velocity ratio (V R.) is defined us the ratio between the distance moved by the effort to the distance moved by the load.

**5.** Mechanical advantage is given as

$$\text{M. A.} = \frac{\text{Weight lifted}}{\text{Effort applied}} = \frac{W}{P}$$

**6.** Efficiency of machine in terms of M.A. and V.R is given as

$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

**7.** The law of a machine is given by

$$P = mW + C$$

where $P$ = Effort applied
$W$ = Weight lifted
$m$ = Slope of the straight line and equal to co-efficient of friction
$C$ = Constant

**8.** A screw-jack is a device used for lifting heavy weight or loads with the help of a small effort applied at its handle.

**9.** Velocity ratio for a differential screw-jack is given by

$$\text{V.R.} = \frac{2\pi L}{(p_1 - p_2)}$$

where $L$ = Length of lower arm
$p_1$ = Pitch of screw $A$
$p_2$ = Pitch of screw $B$.

**10.** The V.R. of wheel and axle is given by

$$\text{V.R.} = \frac{D}{d}$$

where $D$ = Dia. of wheel and $d$ = Dia. of axle.

**11.** The V R. of differential wheel and axle is given by

$$\text{V.R.} = \frac{2D}{(d_1 - d_2)}$$

where $D$ = Dia. of effort wheel
$d_1$ = Dia. of bigger axle
$d_2$ - Dia. of smaller axle.

**12.** The velocity ratio of a purchase crab winch is given by

$$\text{V.R.} = \frac{2D}{D} \times \frac{T_2}{T_1} \text{ ... For a single purchase crab}$$
$$= \frac{2D}{D} \times \frac{T_2}{T_1} \times \frac{T_2}{T_1} \text{ ... For a double purchase crab.}$$

**13.** The V.R. of first system of pulley is given by, V.R. = $2^n$
where $n$ = No. of movable pulleys in the system.

**14.** The V.R. of the 2nd system of pulley is given by V.R. = $n$
where n = No. of segments supporting the load or movable block
= always total number of pulleys in two blocks.

**15.** The velocity ratio of 3rd system of pulley is given by V.R. = $2^n - 1$, where $n$ = No. of pulleys.

## TEST YOURSELF

### (A) Theoretical Problems

**1.** What is the difference between an actual machine and an ideal machine?
**2.** Define the efficiency of a machine. Derive an expression for efficiency of a machine in terms of mechanical advantage and velocity ratio.
**3.** Distinguish between velocity ratio and mechanical advantage. Under what conditions are these two equal.
**4.** What is the law of a machine?
**5.** Choose the correct answers:

(i) If in a machine, velocity ratio is equal to mechanical advantage, then the machine is
(a) Ideal (b) Actual
(c) Any one of the above (d) None of the above
**[Ans.** (a)**]**

(ii) Ideal effort required to run a machine is
(a) More than actual effort (b) Less than actual effort
(c) Equal to actual effort (d) None of the above
**[Ans.** (b)**]**

(iii) If the work is done by the machine in a reserve direction, the machine is known as
(a) Irreversible (b) Self-looking

(c) Reversible (d) None of the above

**[Ans.** (c)**]**

(iv) The law for an ideal machine is given by

(a) $P = mW + C$ (b) $P = mW - C$

(c) $P = mW$ (d) $P = C$

**[Ans.** (c)**]**

**6.** What is a screw jack?

**7.** Define mechanical advantage, velocity ratio and efficiency. *(S – 1989)*

**8.** Derive an expression for the velocity ratio for the following lifting machines:

(a) Wheel and axle, and

(b) Differential wheel and axle,

**9.** Distinguish clearly the difference between the working of a single purchase crab which and a double purchase crab winch.

**10.** What is a pulley? State the working of first system, second system and third system of pulleys. Derive relations of their respective velocity rations.

**(B) Numerical Problems**

**1.** An effort of 50 N is applied to a machine to lift a load of 450 N. The distance moved by the effort is 2.0 m. The load is raised through a distance of 20 cm. Determine the mechanical advantage, velocity ratio and efficiency of the machine. [**Ans.** 9.0, 10.0, 90%]

**2.** The efficiency of a lifting machine is 70%. An effort of 100 N is applied through a distance of 3 m to the lifting machine raise a load through a distance of 30 cm. Determine: (i) Load lifted by the machine, (ii) Mechanical advantage, and (iii) Velocity ratio.

[**Ans.** (i) 700 N. (ii) 7.0, (iii) 10.0]

**3.** An effort of 40 N is applied to a machine to lift a load of 1800 N. The distance moved by effort is 3.60 m and by the load the distance moved in 6 cm. Determine: (i) Mechanical advantage of the machine, (a) Velocity ratio of the machine, (iii) Efficiency, and (iv) Ideal effort required.

[**Ans.** (i) 45, (ii) 60.0, (iii) η -75% and (iv) 30 N]

**4.** Find the law of a machine in which an effort of 11.6 N raised a load of 50 N and an effort of 17.6 N raised a load of 8O N. Find what effort is required to lift a load of 70 N? [**Ans.** $P = 0.2\ W + 1.6$ ; 15.6 N]

**5.** The velocity ratio of a machine is 10 and efficiency is 80%. Determine the effort required to lift a load of 100 N. What in the law of the machine'.

[**Ans.** 12.5 N, $P = 0.1\ W + 2.5$]

**6.** A load of 4000 N is to be lifted by a screw-jack, having threads of 10 mm pitch. The efficiency of the jack at this load is 40%. Determine the effort applied at the end of a handle of 60 cm length. [**Ans.** 26.5 N]

**7.** In a differential screw-jack, the screw-threads have pitch of 12 mm and 9 mm. If the efficiency of the machine is 30%, find the effort required at the end of the arm 40 cm long to lift a load of 4 kN. [**Ans.** 15.9 KN]

**8.** A weight of 460 N is to be raised by means of a wheel and axle. The axle is 10 cm diameter and wheel is 40 cm diameter. If a force of 120 N has to be applied to the wheel, find: (i) M.A., (ii) V.R. and (iii) efficiency of the machine.

[**Ans.**(i) 3.83, (in) 4.0, (iii) 95.75%]

**9.** For a differential wheel and axle, the diameter of the wheel is 24 cm. The larger and smaller diameters of the differential axle are 8 cm and 7 cm respectively. An effort of 320 N is applied to lift a load of 8 kN. Determine: (i) velocity ratio, (ii) mechanical advantage, and (iii) efficiency of this machine. [**Ans.**(i) 48, (ii) 25 and (iii) 62.0846]

**10.** The followings are the specifications of a single purchase crab:

Diameter of load drum = 20 cm
Length of lever = 120 cm
No. of teeth on pinion = 10
No. of teeth on spur wheel = 100.

Find the velocity ratio of the machine. On this machine efforts of 100 N and 160 N are required to lift loads of 3 kN and 9 kN respectively. Find the law of the machine and the efficiencies at the above loads.

[**Ans.** 120, $P = 0.01\ W + 7$, $\eta$ = 75%]

**11.** In a double purchase crab, the pinions have 16 and 20 teeth, while the spur wheels have 45 and 40 teeth. The effort handle is 40 cm long while the effective diameter of the drum is 15 cm. If the efficiencies of the winch is 40%, what load will be lifted by an effort of 250 N applied at the end of the handle. [**Ans.** 3.2 kN]

**12.** In a first system of pulleys there are four movable pulleys. If an effort of 100 N lifts a load of 1360 N, find : (a) effort wasted in friction, (b) the load wasted in friction. [**Ans.**(a) 15 N. (6) 240 N]

**13.** In a second system of pulleys there are three pulleys in the upper block and two pulleys in the lower block. If the efficiency of the pulley system is 75%, find the effort required to lift a load of 1000 N.

[**Ans.** 266.67 N]

**14.** There are 4 pulleys arranged in the third system of pulleys. Find the effort required to lift a load of 1.8 kN, if the efficiency of the machine is 75%. [**Ans.** 1.6 kN]